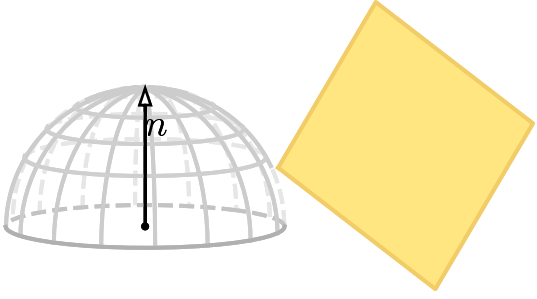
SIGGRAPH 2021

BRDF IMPORTANCE SAMPLING FOR POLYGONAL LIGHTS

CHRISTOPH PETERS
KARLSRUHE INSTITUTE OF TECHNOLOGY

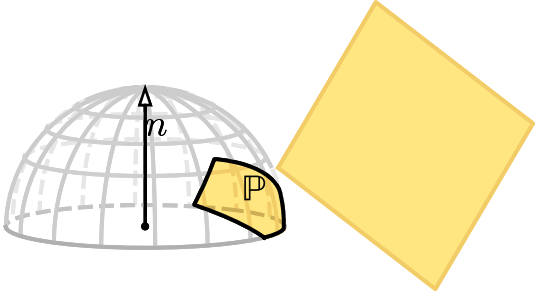
Area sampling

$$L_o(\omega_o) = \int_\Omega L_i(\omega_i) f_r(\omega_i, \omega_o) n^\mathsf{T} \omega_i \, \mathrm{d}\omega_i$$

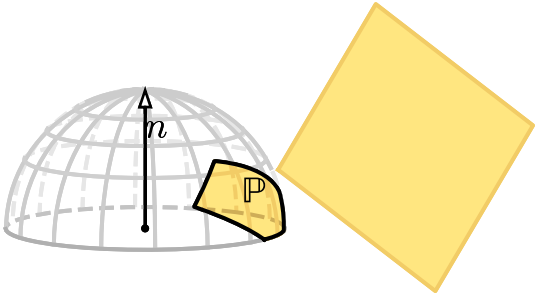


Area sampling

$$L_o(\omega_o) = L_e \int_{\mathbb{P}} V(\omega_i) f_r(\omega_i, \omega_o) n^{\mathsf{T}} \omega_i \, \mathrm{d}\omega_i$$



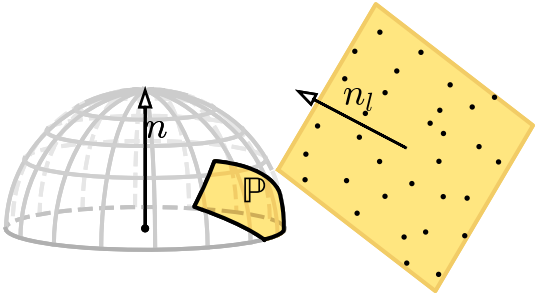
Area sampling



$$L_o(\omega_o) = L_e \int_{\mathbb{P}} V(\omega_i) f_r(\omega_i, \omega_o) n^\mathsf{T} \omega_i \, \mathrm{d}\omega_i$$

$$L_o(\omega_o) \approx L_e V(\omega_i) f_r(\omega_i, \omega_o) \frac{n^\mathsf{T} \omega_i}{p(\omega_i)}$$

Area sampling



$$L_o(\omega_o) = L_e \int_{\mathbb{P}} V(\omega_i) f_r(\omega_i, \omega_o) n^\mathsf{T} \omega_i \, \mathrm{d}\omega_i$$

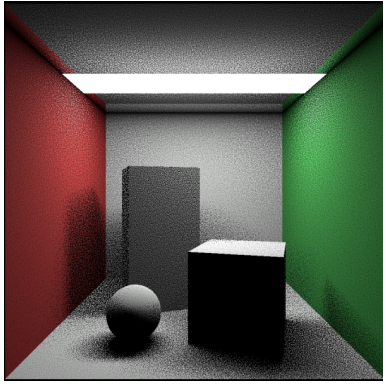
$$L_o(\omega_o) \approx L_e V(\omega_i) f_r(\omega_i, \omega_o) \frac{n^\mathsf{T} \omega_i}{p(\omega_i)}$$

$$= L_e V(\omega_i) f_r(\omega_i, \omega_o) \frac{n^\mathsf{T} \omega_i |n_l^\mathsf{T} \omega_i|}{\|x - y\|^2} A$$

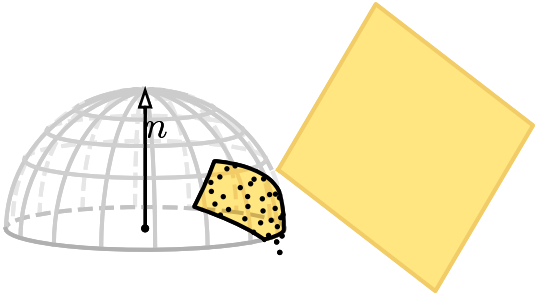
$A :=$ area

Result at 1 sample per pixel

Area



Solid angle sampling



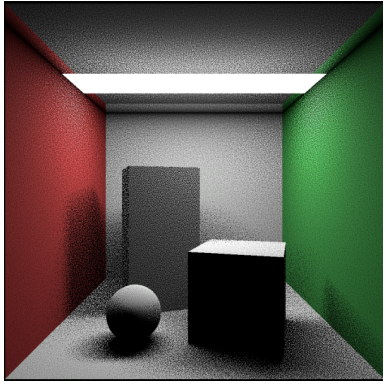
$$L_o(\omega_o) = L_e \int_{\mathbb{P}} V(\omega_i) f_r(\omega_i, \omega_o) n^\mathsf{T} \omega_i \, \mathrm{d}\omega_i$$

$$L_o(\omega_o) \approx L_e V(\omega_i) f_r(\omega_i, \omega_o) \, n^\mathsf{T} \omega_i A$$

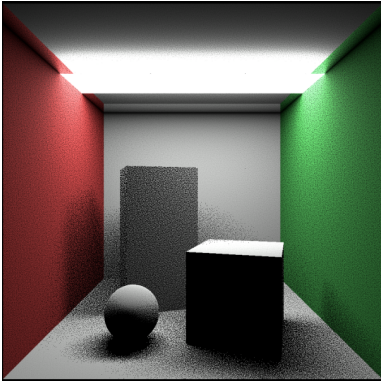
$A :=$ solid angle

Results at 1 sample per pixel

Area

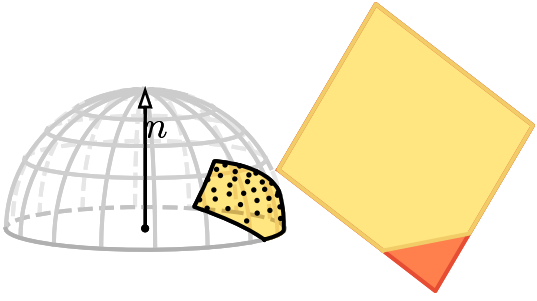


Solid angle



Projected solid angle sampling (ours)

$$L_o(\omega_o) = L_e \int_{\mathbb{P}} V(\omega_i) f_r(\omega_i, \omega_o) n^\mathsf{T} \omega_i \, \mathrm{d}\omega_i$$

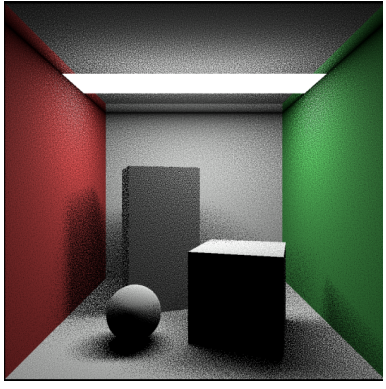


$$L_o(\omega_o) \approx L_e V(\omega_i) f_r(\omega_i, \omega_o) A$$

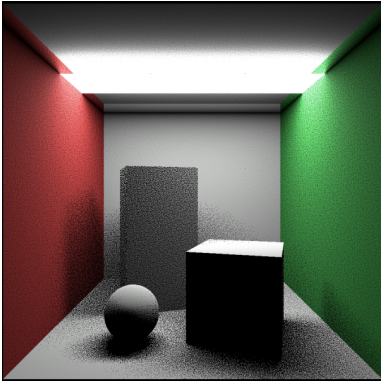
$A :=$ projected solid angle

Results at 1 sample per pixel

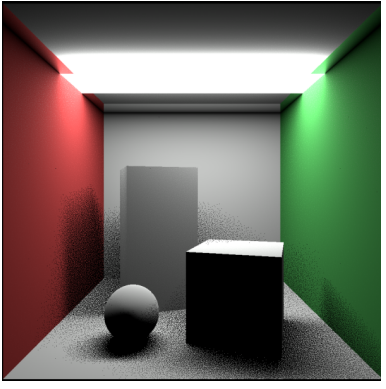
Area



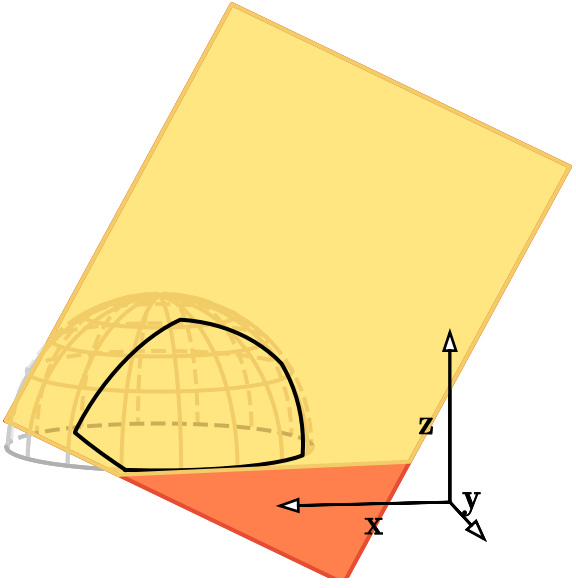
Solid angle



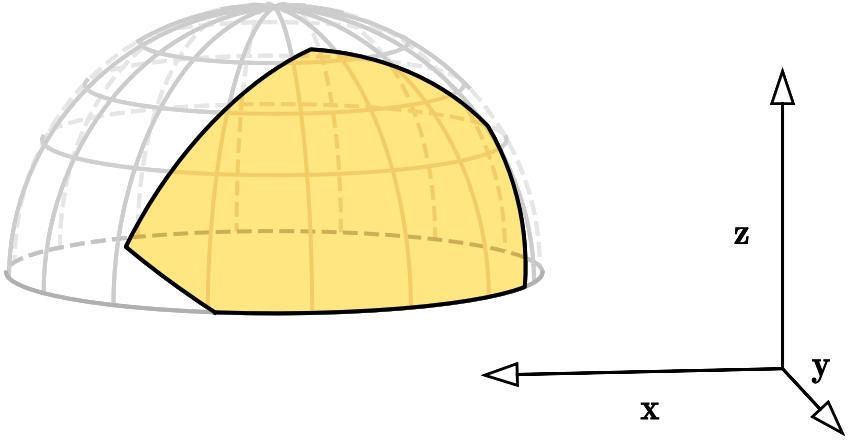
Projected solid angle



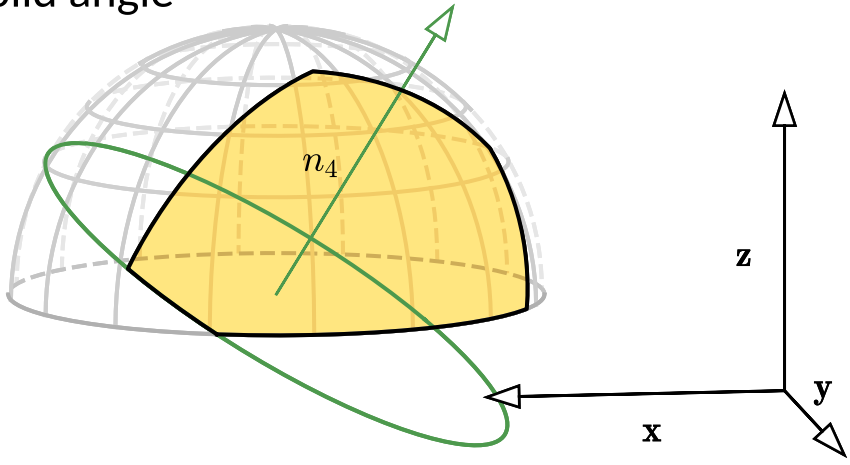
Projected solid angle



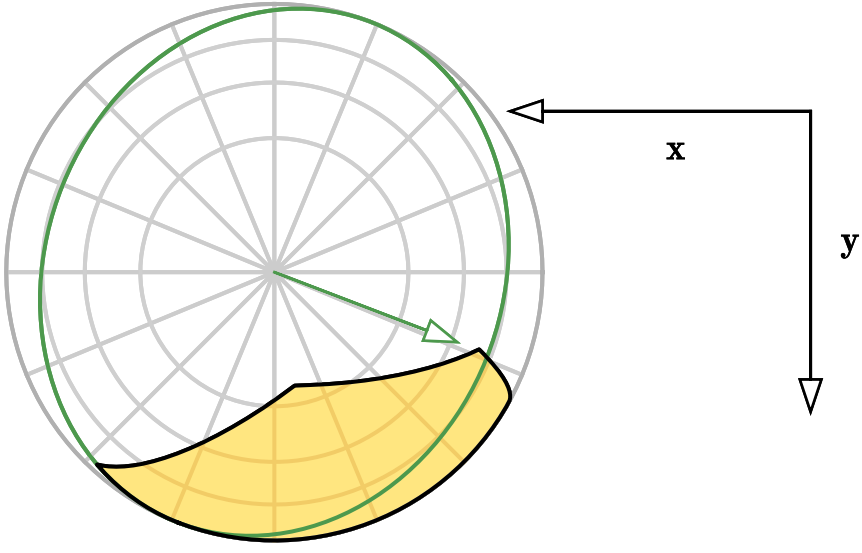
Projected solid angle



Projected solid angle



Projected solid angle



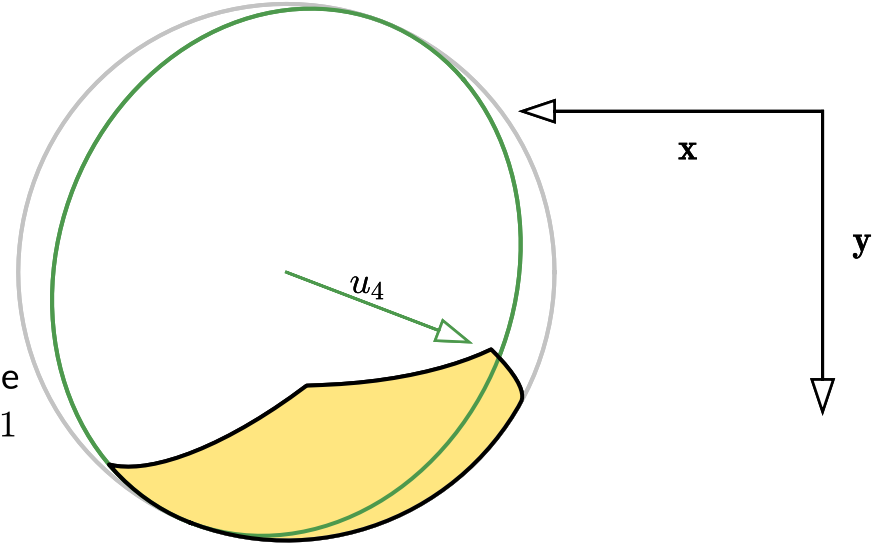
Projected solid angle

$u_j := \frac{n_{j,\mathrm{xy}}}{n_{j,\mathrm{z}}} \in \mathbb{R}^2$

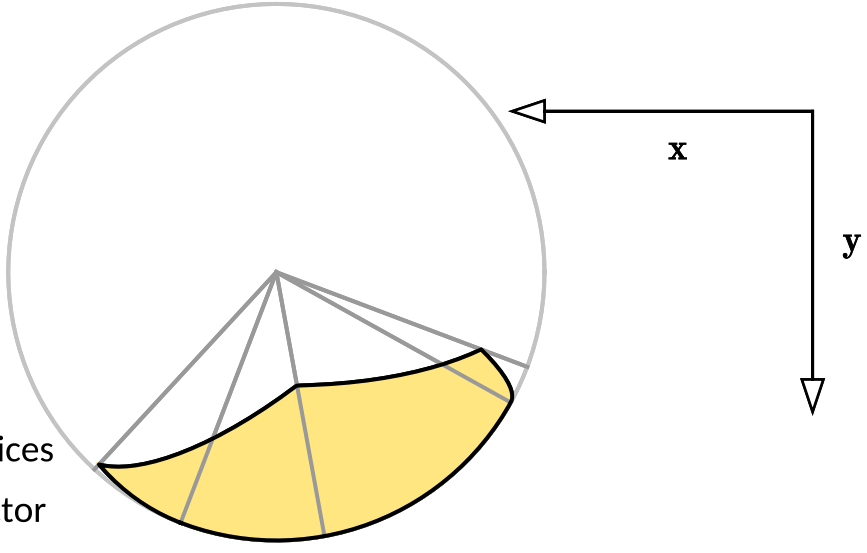
$C_j := I + u_j u_j^\mathsf{T}$

$q \in \mathbb{R}^2$ in ellipse

$\Leftrightarrow \quad q^\mathsf{T} C_j q \leq 1$



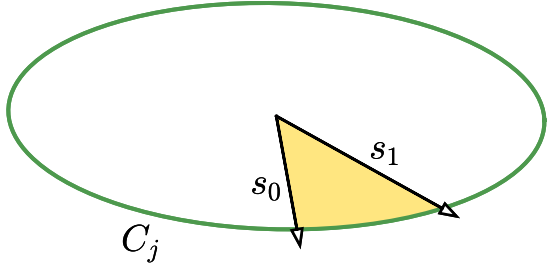
Projected solid angle



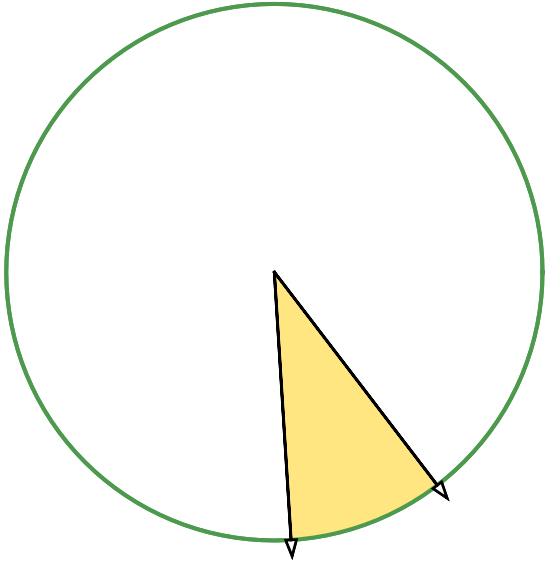
Cut through vertices

2 ellipses per sector

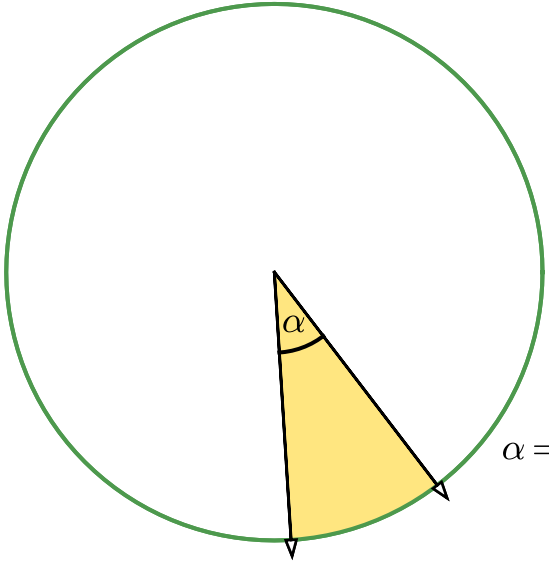
Projected solid angle



Projected solid angle

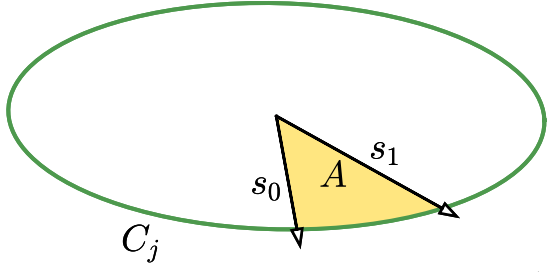


Projected solid angle



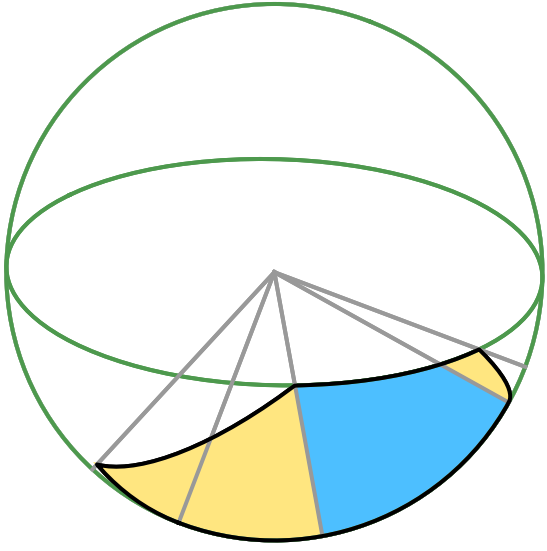
$$\alpha = \operatorname{atan2}\left(\begin{matrix} -|(s_0, s_1)| \\ \frac{1}{|C_j|} s_0^\top C_j s_1 \end{matrix}\right)$$

Projected solid angle

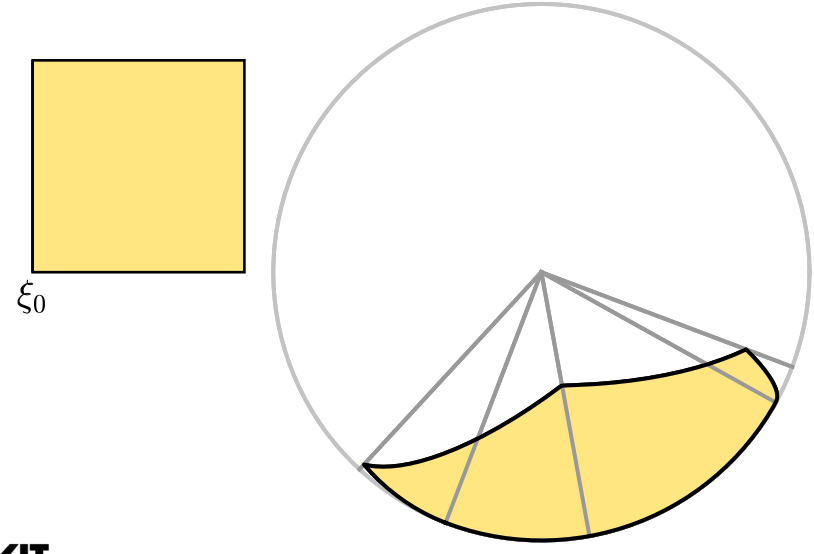


$$A = \frac{1}{2|C_j|}\operatorname{atan2}\left(\begin{matrix}-|(s_0, s_1)| \\ \frac{1}{|C_j|} s_0^\mathsf{T} C_j s_1\end{matrix}\right)$$

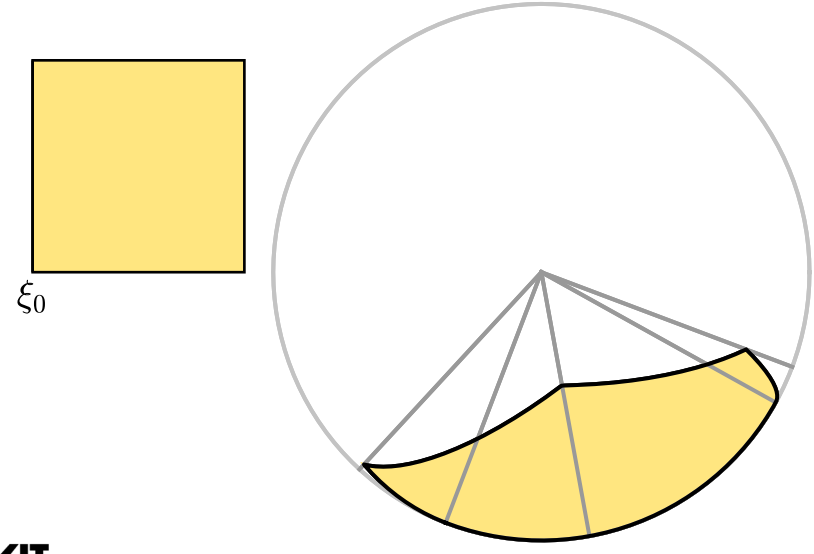
Projected solid angle



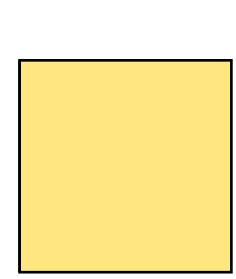
Sampling the projected solid angle



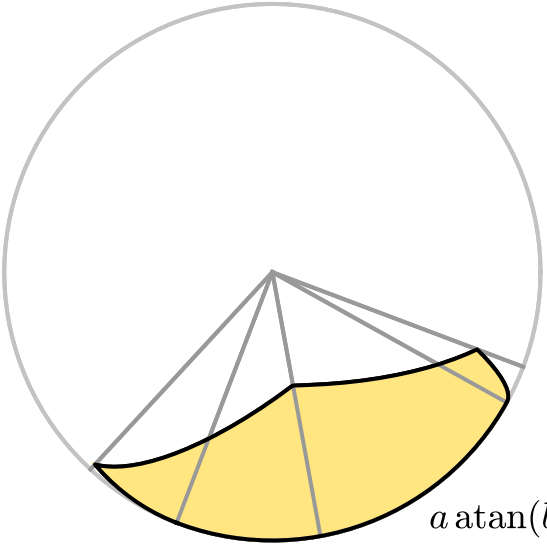
Sampling the projected solid angle



Sampling the projected solid angle

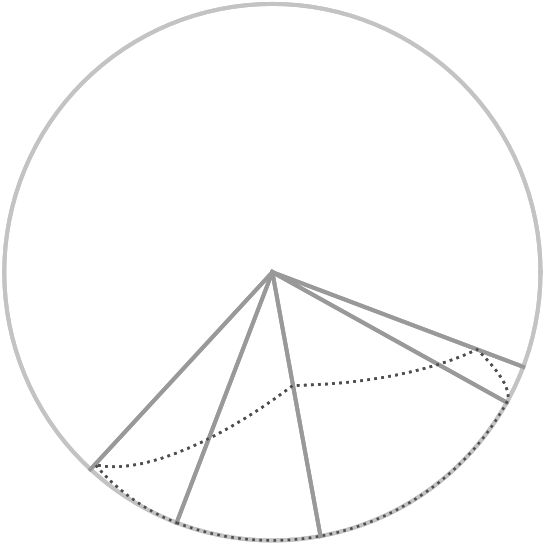


ξ_0



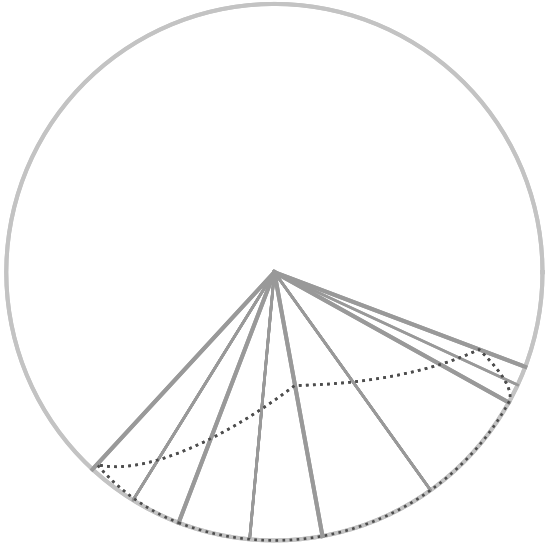
$$a\,\mathrm{atan}(bx) + c\,\mathrm{atan}(dx) = \xi_0$$

Initialization



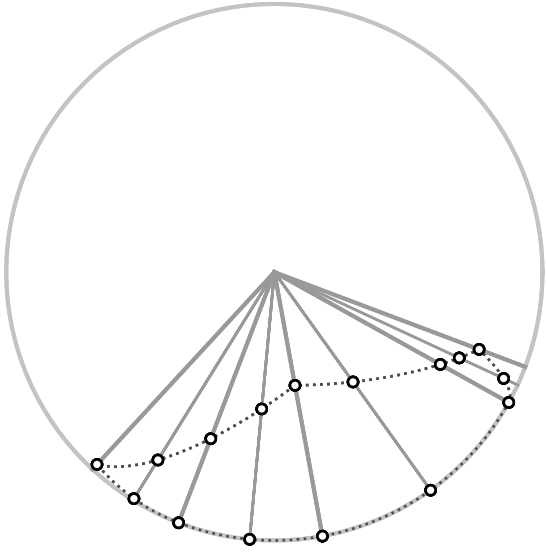
Initialization

Halve sectors



Initialization

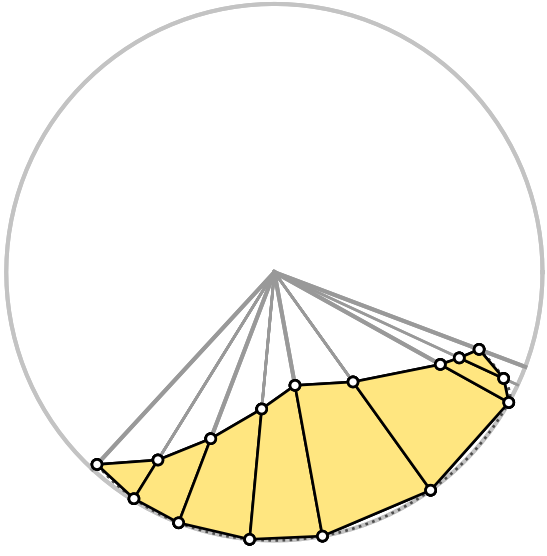
Halve sectors



Initialization

Halve sectors

Form quads

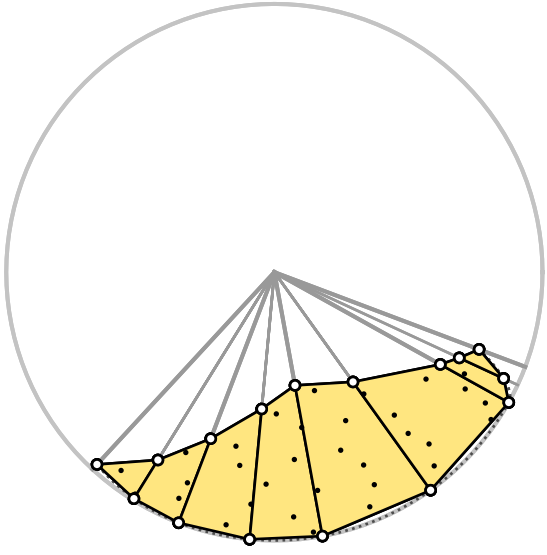


Initialization

Halve sectors

Form quads

Sample quads

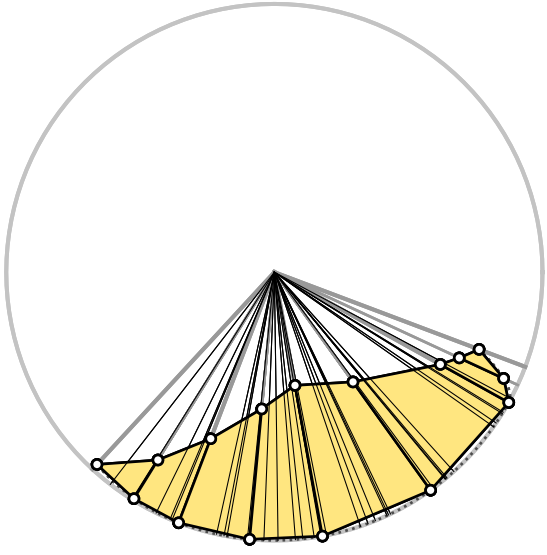


Initialization

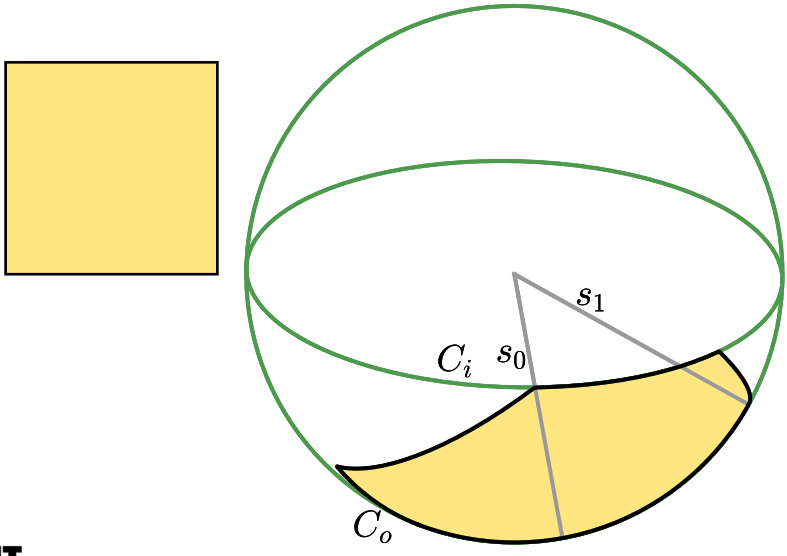
Halve sectors

Form quads

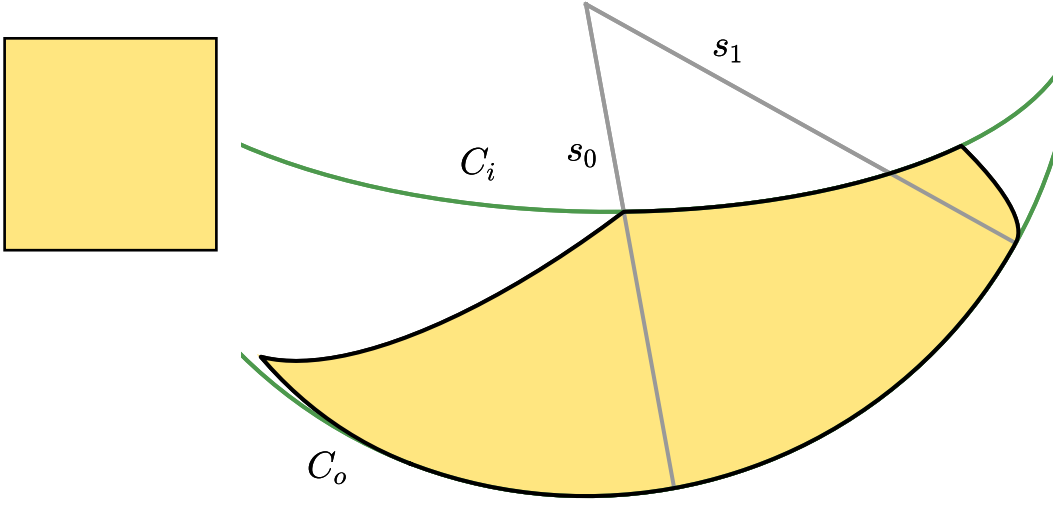
Sample quads



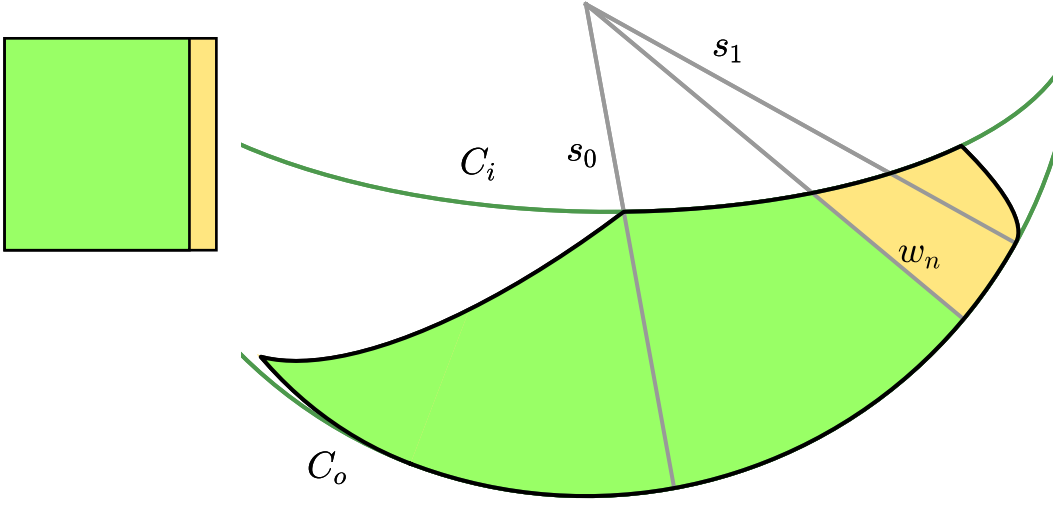
Iteration



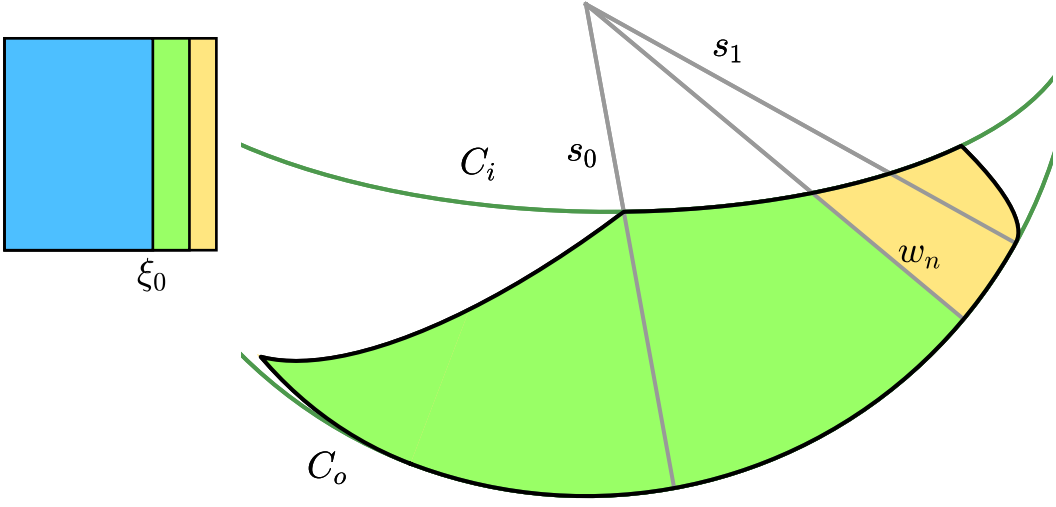
Iteration



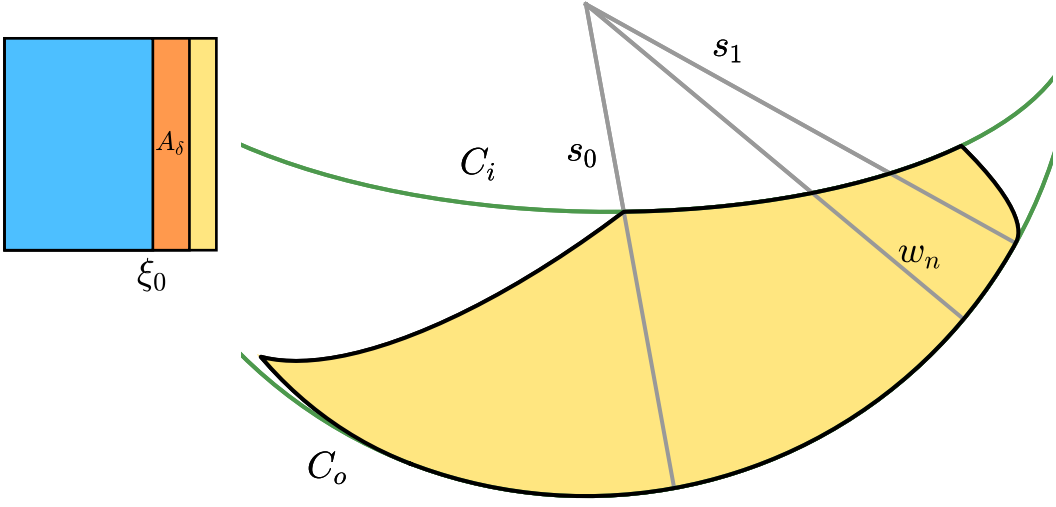
Iteration



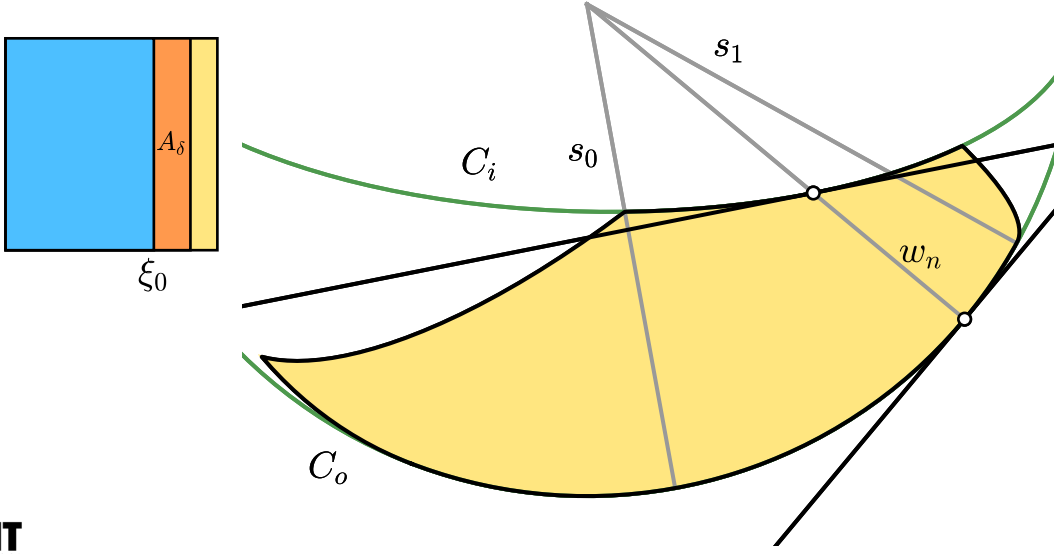
Iteration



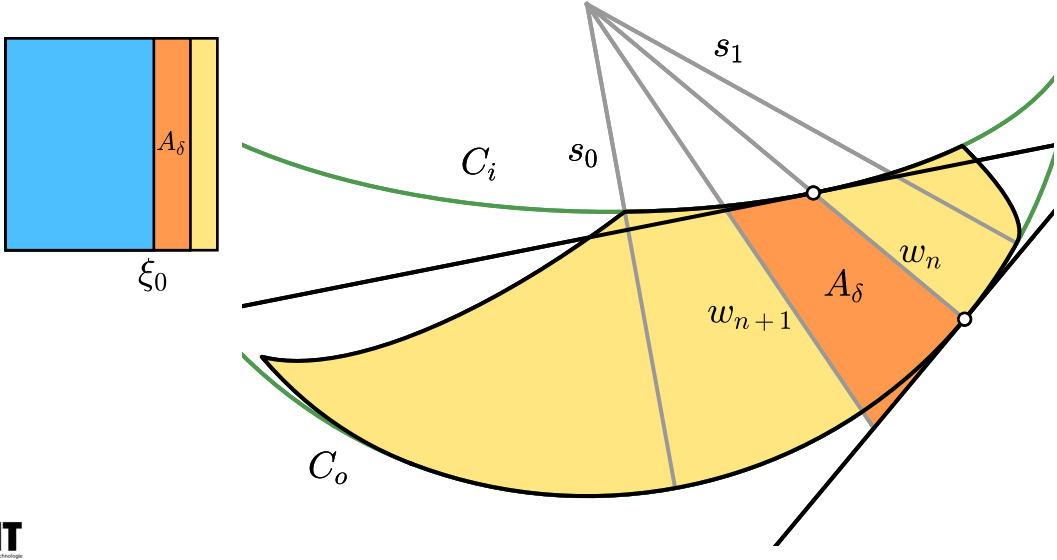
Iteration



Iteration



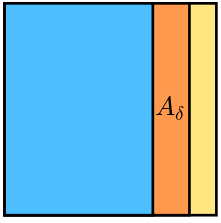
Iteration

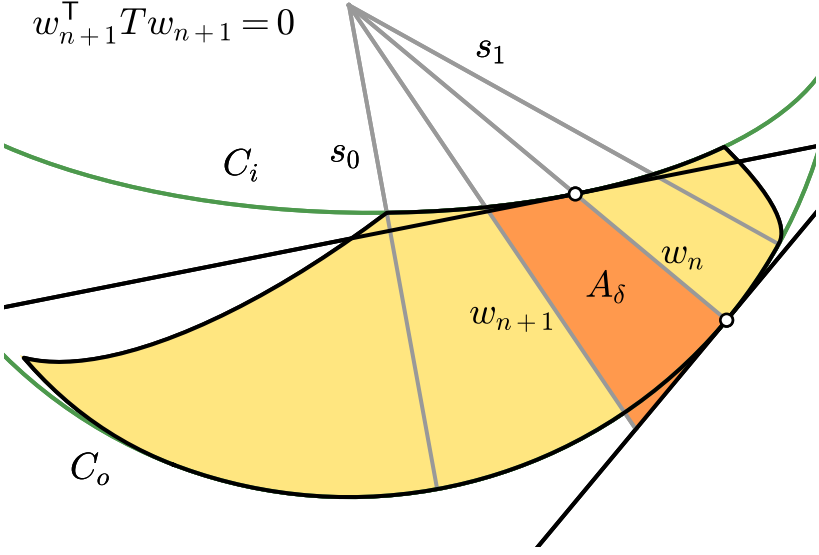


Iteration

$$T := \begin{pmatrix} -w_{n,y} \\ w_{n,x} \end{pmatrix} w_n^\mathsf{T}(C_i - C_o) - 2A_\delta C_i w_n w_n^\mathsf{T} C_o \in \mathbb{R}^{2\times 2}$$

$$w_{n+1}^\mathsf{T} T w_{n+1} = 0$$





Error analysis

Fix iteration count to 2

Worst case error in ξ_0: $1.8 \cdot 10^{-5}$

99th percentile of error in ξ_0: $4.6 \cdot 10^{-15}$

Error analysis

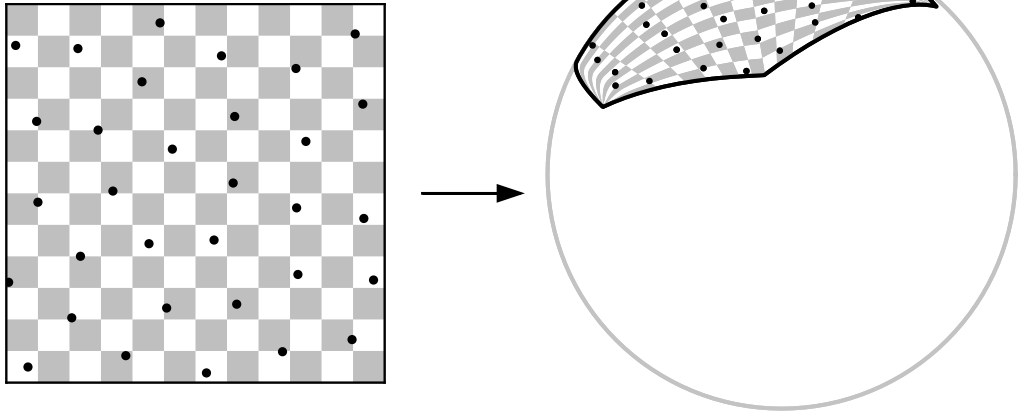
Fix iteration count to 2

Worst case error in ξ_0: $1.8 \cdot 10^{-5}$

99th percentile of error in ξ_0: $4.6 \cdot 10^{-15}$

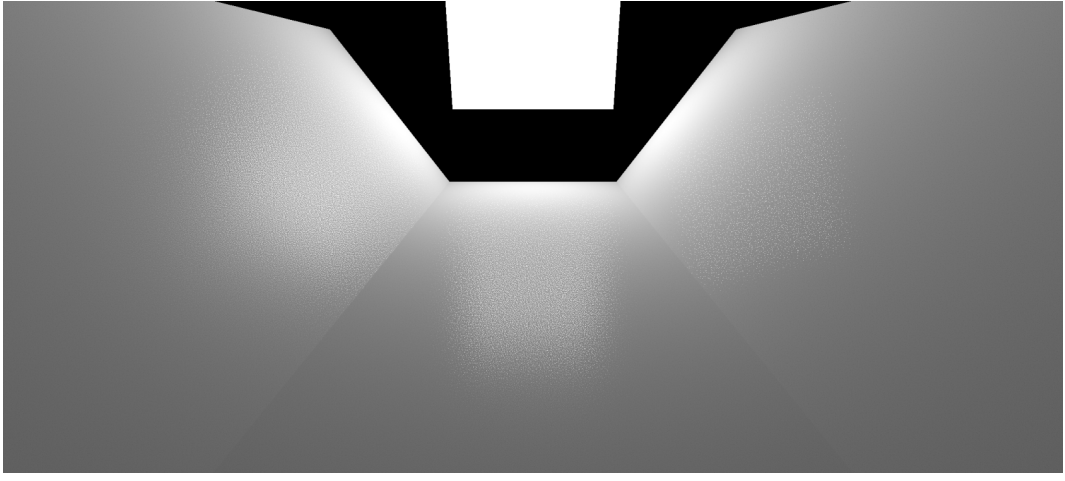
$\rightarrow$ unbiased

Sample mapping



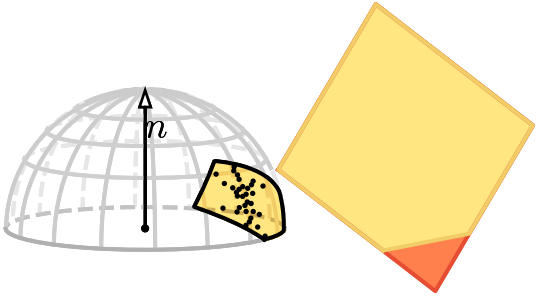
Planes of varying roughness (1 spp)

Projected solid angle sampling



LTC importance sampling

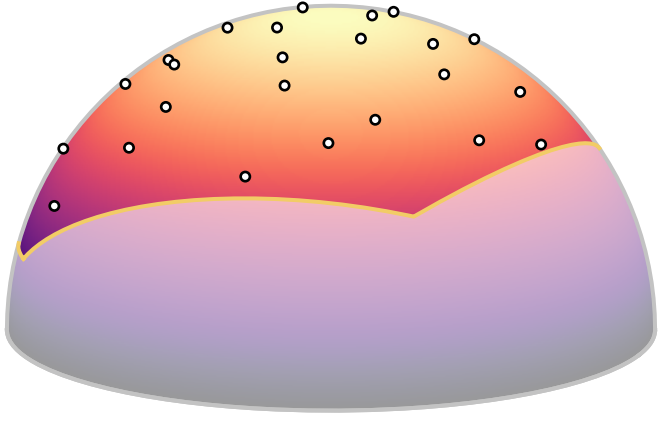
$$L_o(\omega_o) = L_e \int_{\mathbb{P}} V(\omega_i) f_r(\omega_i, \omega_o) n^{\mathsf{T}} \omega_i \, \mathrm{d}\omega_i$$



$$L_o(\omega_o) \approx V(\omega_i) A$$

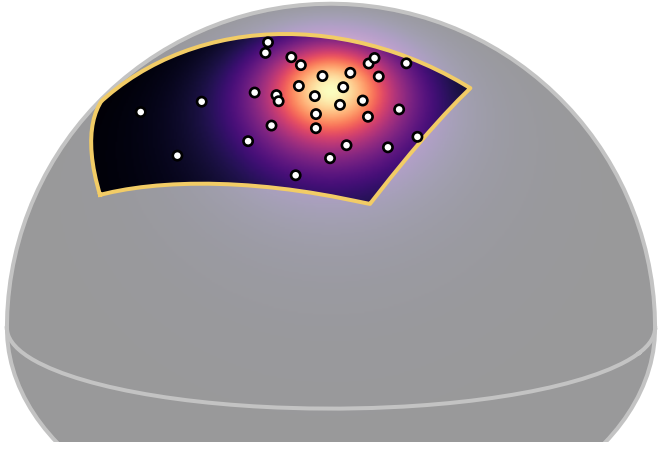
$A :=$ unshadowed shading

Linearly transformed cosines [Heitz et al. 2016]



cosine distribution

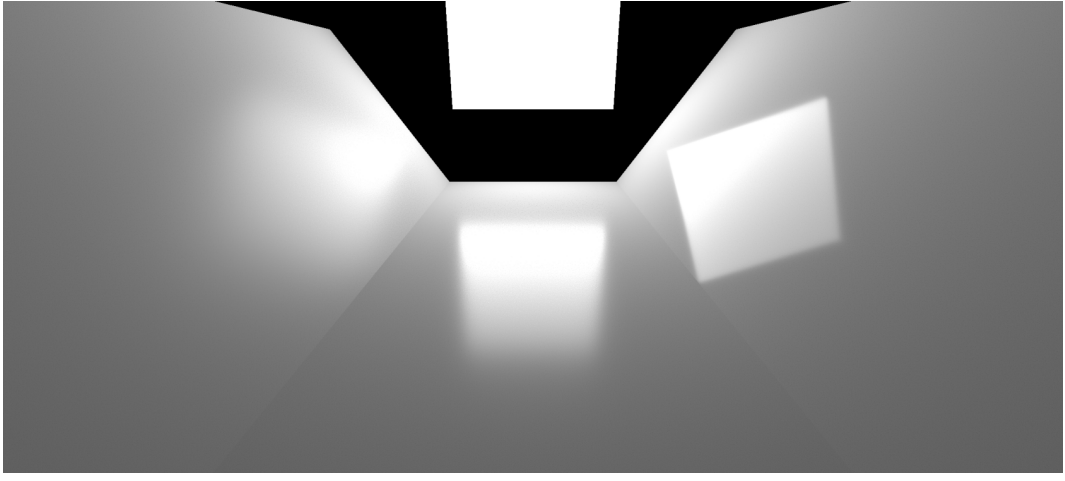
Linearly transformed cosines [Heitz et al. 2016]



linearly transformed cosine distribution

Planes of varying roughness (2 spp)

MIS: LTC + projected solid angle sampling



Numerical stability

Conclusions

Iterative methods on GPU: Better than their reputation

Conclusions

Iterative methods on GPU: Better than their reputation

Stability matters

Conclusions

Iterative methods on GPU: Better than their reputation

Stability matters

Put it into your renderer now

Code at https://momentsingraphics.de/Siggraph2021.html

Conclusions

Iterative methods on GPU: Better than their reputation

Stability matters

Put it into your renderer now

Code at https://momentsingraphics.de/Siggraph2021.html

Thanks!