BRDF Importance Sampling for Linear Lights

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Karlsruhe Institute of Technology

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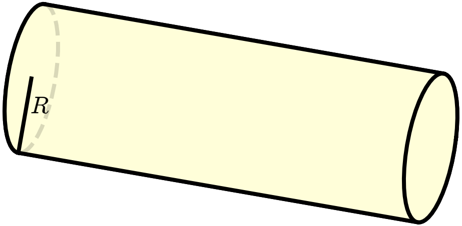
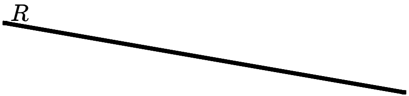
Resident Evil 7 (© Capcom)

Linear light := infinitesimally thin cylinder



$$R \to 0$$

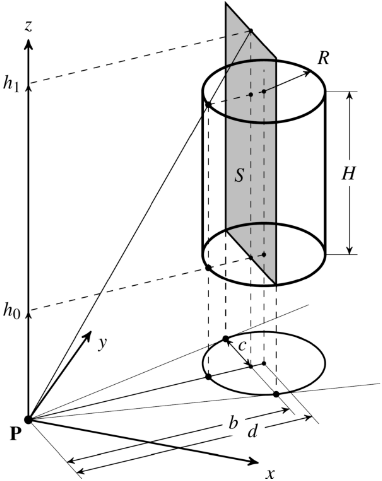
Linear light := infinitesimally thin cylinder

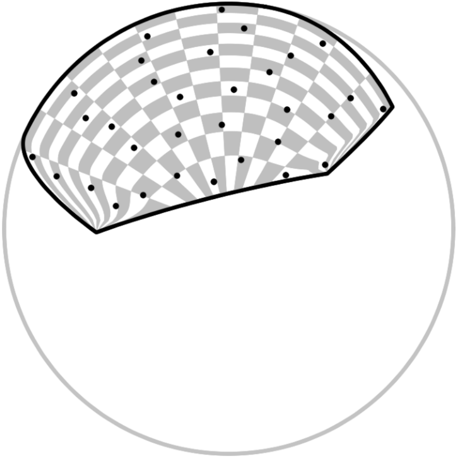


$$R \to 0$$

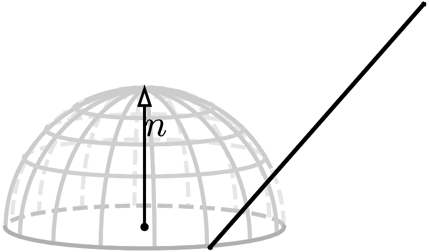
BRDF importance sampling for cylindrical lights

Gamito 2016 + Peters 2021



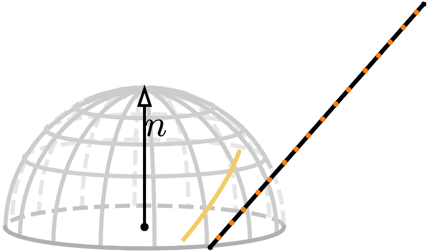


Area sampling



$$L_o(\omega_o) = \int_\Omega L_i(\omega_i) f_r(\omega_i, \omega_o)\, n \cdot \omega_i \,\mathrm{d}\omega_i$$

Area sampling



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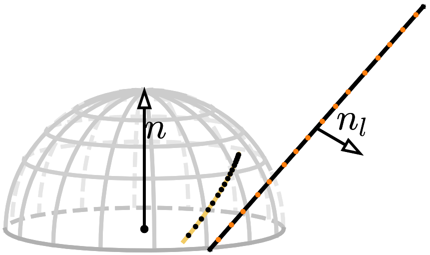
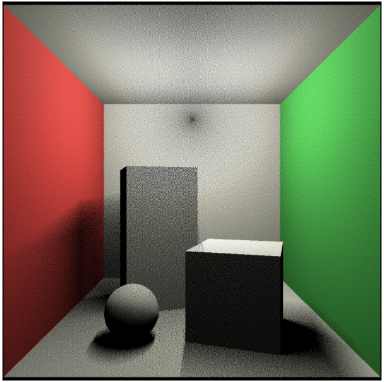


$$L_o(\omega_o) = \int_\Omega L_i(\omega_i) f_r(\omega_i, \omega_o)\, n \cdot \omega_i \, \mathrm{d}\omega_i$$

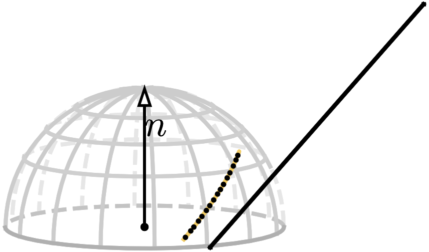
$$p(\omega_i) \propto L_e(\omega_i) \frac{\|x-y\|^2}{n_l \cdot \omega_i}$$

Result at 1 sample per pixel

Area



Solid angle sampling



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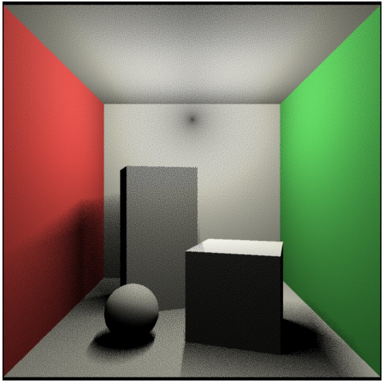
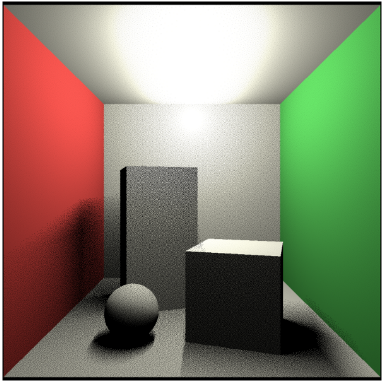
Results at 1 sample per pixel

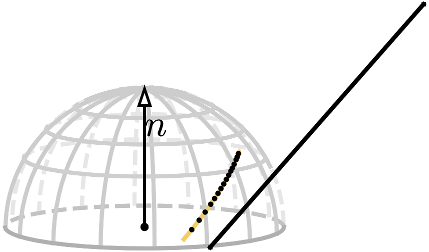
Area



Solid angle



Projected solid angle sampling (ours)



$$L_o(\omega_o) = \int_\Omega L_i(\omega_i) f_r(\omega_i, \omega_o)\, n \cdot \omega_i \,\mathrm{d}\omega_i$$

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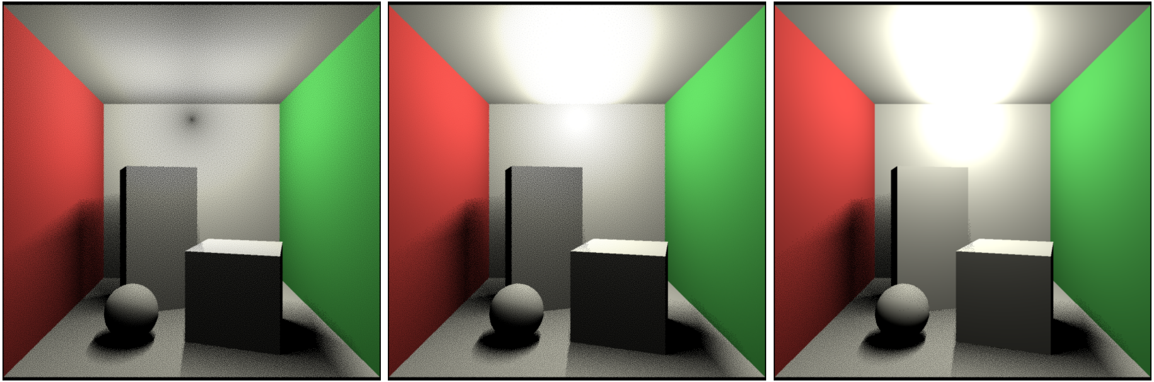
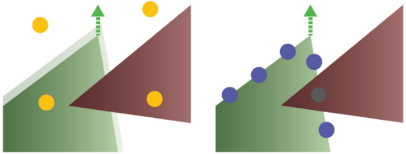
Results at 1 sample per pixel

Area	Solid angle	Projected solid angle



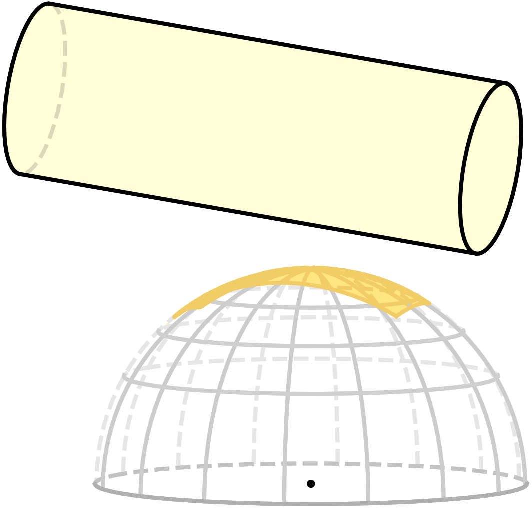
Projected solid angle sampling [Li et al. 2018]



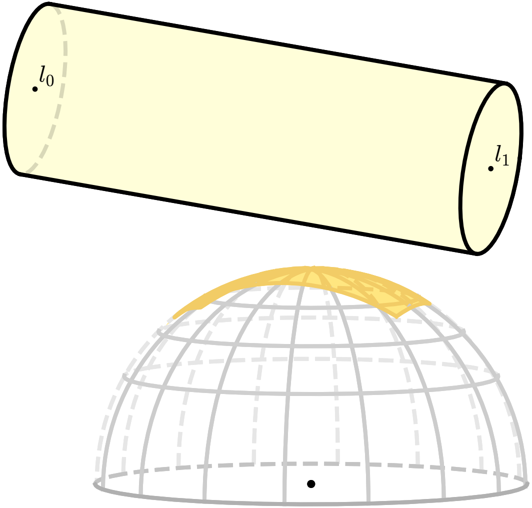
(a) area sampling (b) edge sampling

```
for (uint i = 0; i != 20; ++i) {
    bool no_bisection = (current >= left && current <= right);
    current = no_bisection ? current : (0.5f * (left + right));
    float error = get_line_sampling_cdf_li(line, current) - random_number;
    if (abs(error) < 1.0e-5f || i == 19) break;
    left = (error > 0.0f) ? left : current;
    right = (error > 0.0f) ? current : right;
    float derivative = get_line_sampling_pdf_li(line, current);
    current -= error / derivative;
}
```

The solid angle of a cylinder



The solid angle of a cylinder



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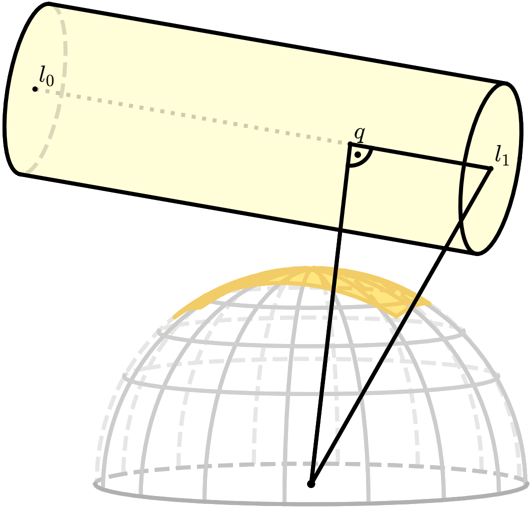

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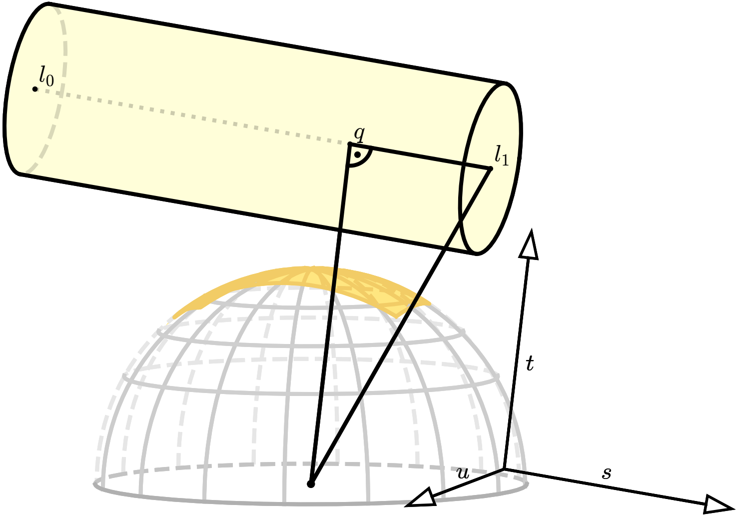

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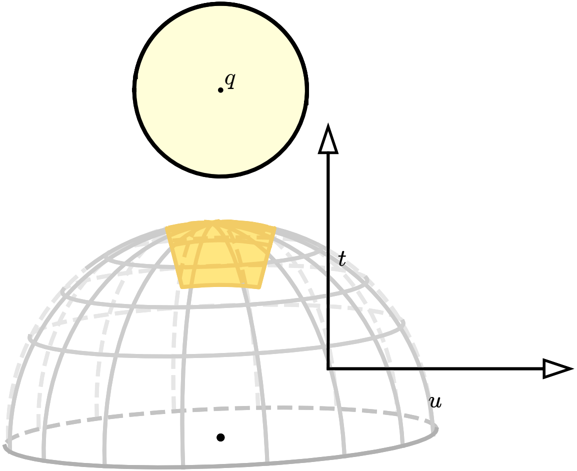
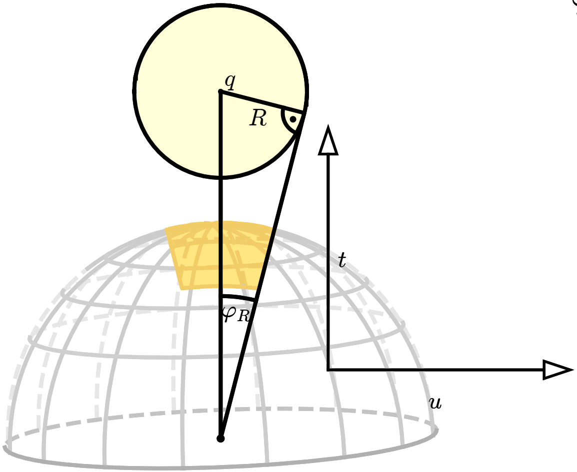


The solid angle of a cylinder



$$\varphi_R := \arcsin \frac{R}{\|q\|}$$

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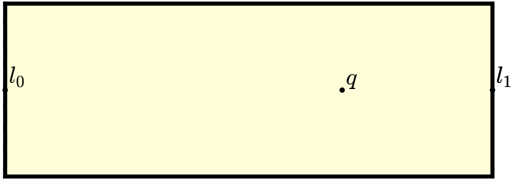

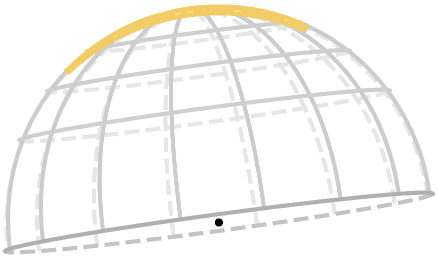

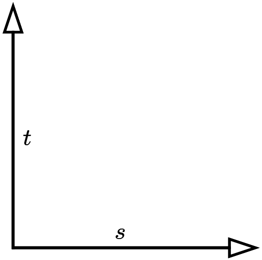

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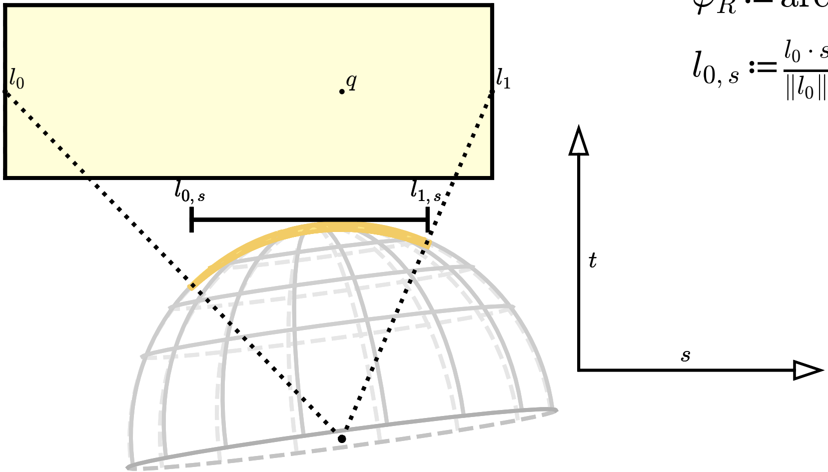
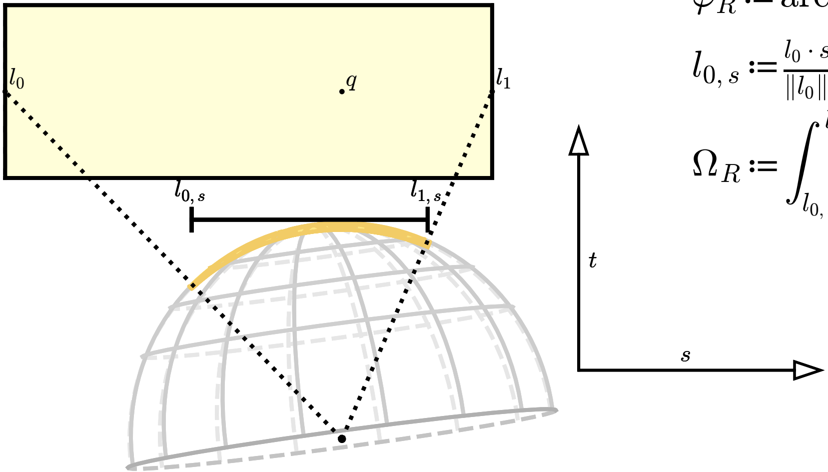


$$\varphi_R := \arcsin \frac{R}{\|q\|}$$

$$l_{0,s} := \frac{l_0 \cdot s}{\|l_0\|}, \quad l_{1,s} := \frac{l_1 \cdot s}{\|l_1\|}$$

The solid angle of a cylinder

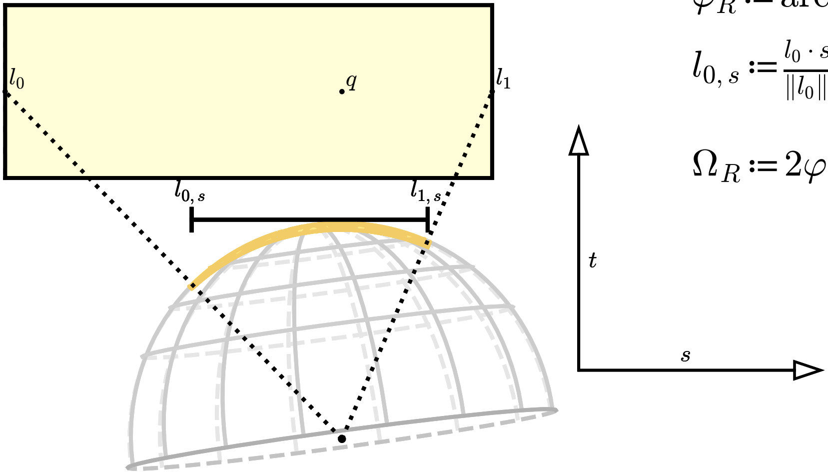


$$\varphi_R := \arcsin \frac{R}{\|q\|}$$

$$l_{0,\,s} := \frac{l_0 \cdot s}{\|l_0\|}, \quad l_{1,\,s} := \frac{l_1 \cdot s}{\|l_1\|}$$

$$\Omega_R := \int_{l_{0,\,s}}^{l_{1,\,s}} \int_{-\varphi_R}^{\varphi_R} 1 \, \mathrm{d}\varphi \, \mathrm{d}\omega_s$$

The solid angle of a cylinder



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$$\Omega_R := 2\varphi_R \int_{l_{0,s}}^{l_{1,s}} 1 \, \mathrm{d}\omega_s$$

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The projected solid angle of a line

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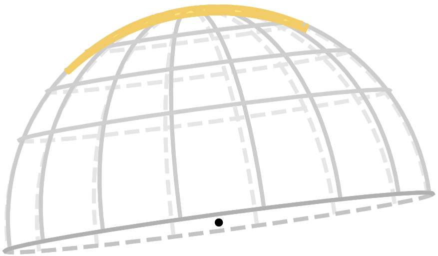
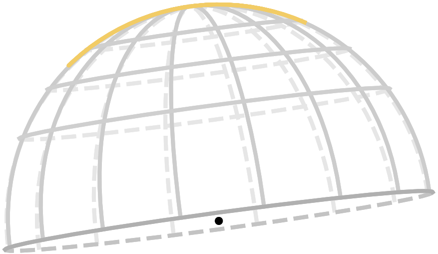


The projected solid angle of a line

$$\Omega_R := 2\varphi_R \int_{l_{0,\,s}}^{l_{1,\,s}} 1 \, \mathrm{d}\omega_s$$

$$\frac{\mathrm{d}\Omega_R}{\mathrm{d}R} := \lim_{R \to 0} \frac{\Omega_R}{R} = \frac{2}{\|q\|} \int_{l_{0,\,s}}^{l_{1,\,s}} 1 \, \mathrm{d}\omega_s$$



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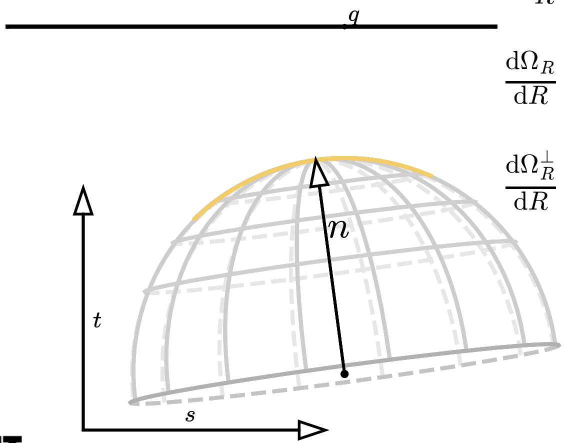

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$$\frac{\mathrm{d}\Omega_R^{\perp}}{\mathrm{d}R} := \frac{2}{\|q\|} \int_{l_{0,s}}^{l_{1,s}} n \cdot (\omega_s s + \sqrt{1-\omega_s^2}\, t) \,\mathrm{d}\omega_s$$

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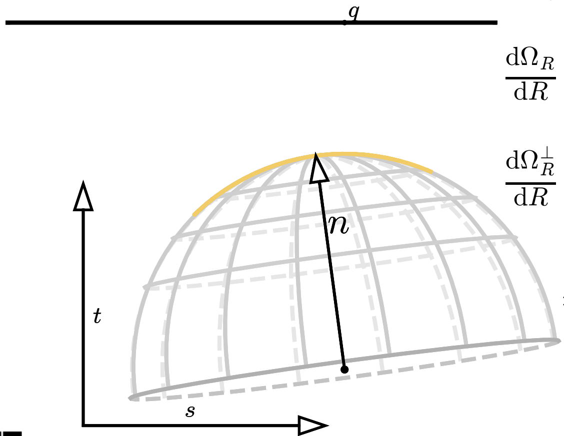

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$$= \frac{2}{\|q\|} \int_{l_{0,s}}^{l_{1,s}} n_s \omega_s + n_t \sqrt{1 - \omega_s^2} \,\mathrm{d}\omega_s$$

Cumulative distribution function

$$F_{n_s,\,n_t}(\omega_s) := 2\int_0^{\omega_s} n_s\omega_s' + n_t\sqrt{1-\omega_s'^2}\,\mathrm{d}\omega_s'$$

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Cumulative distribution function

$$
\begin{aligned}
F_{n_s,\,n_t}(\omega_s) &\coloneqq 2\int_0^{\omega_s} n_s\omega_s' + n_t\sqrt{1-\omega_s'^2}\,\mathrm{d}\omega_s' \\
&= n_s\omega_s^2 + n_t(\sqrt{1-\omega_s^2}\,\omega_s + \arcsin\omega_s)
\end{aligned}
$$

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Cumulative distribution function

$$F_{n_s,n_t}(\omega_s) := 2\int_0^{\omega_s} n_s\omega_s' + n_t\sqrt{1-\omega_s'^2}\,\mathrm{d}\omega_s'$$
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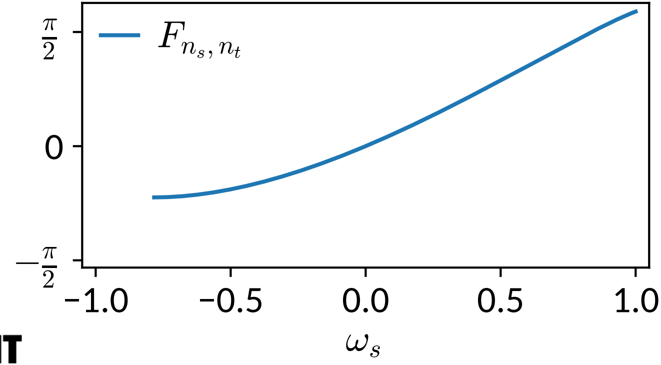

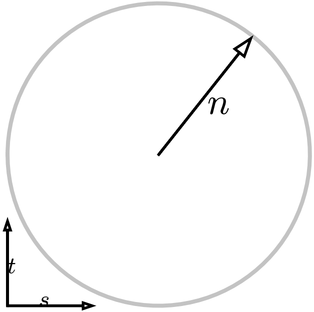

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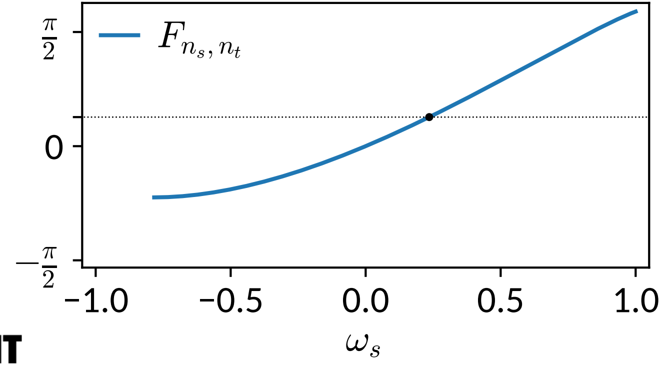

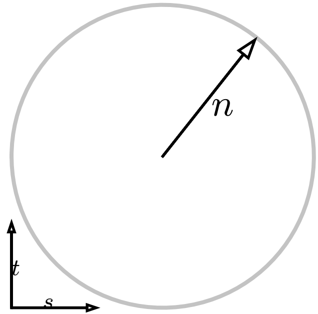

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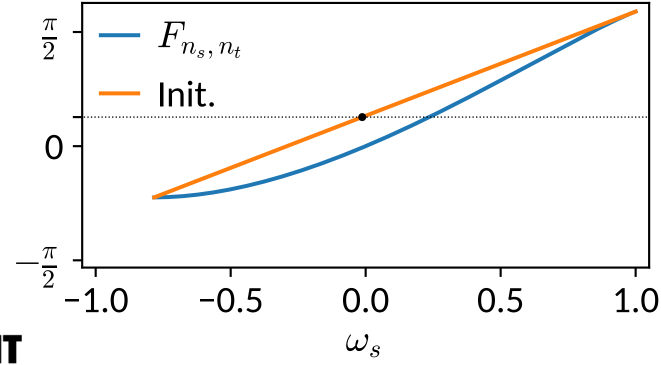
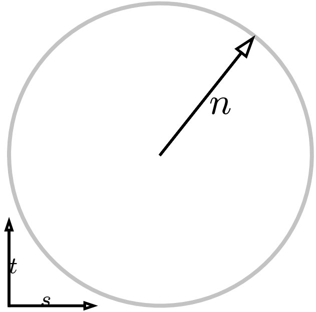
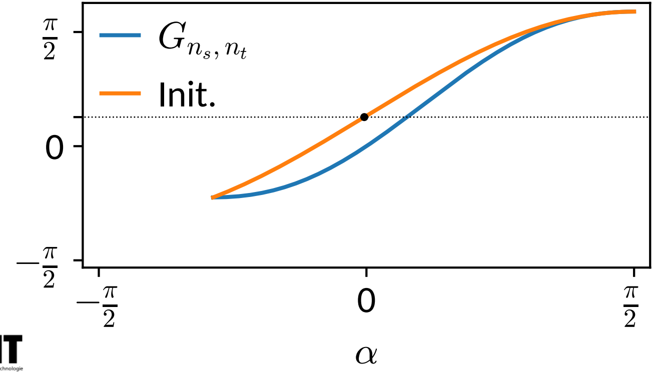


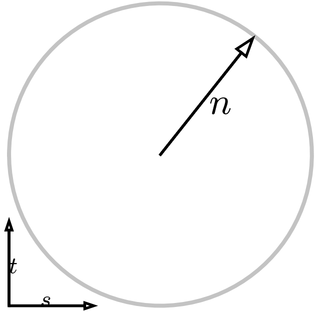


Cumulative distribution function

$\alpha := \arcsin \omega_s$

$G_{n_s,n_t}(\alpha) := F_{n_s,n_t}(\sin\alpha) = n_s \sin^2\alpha + n_t(\cos\alpha \sin\alpha + \alpha)$





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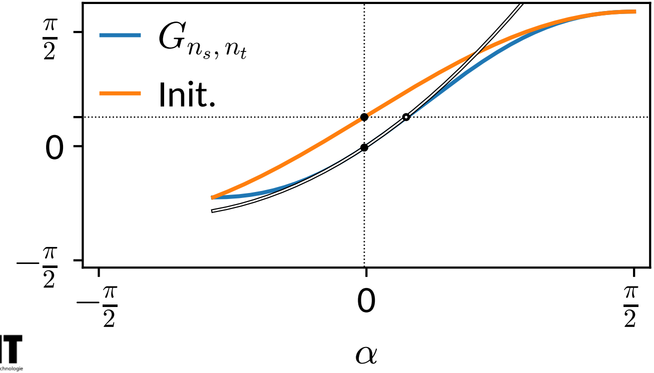

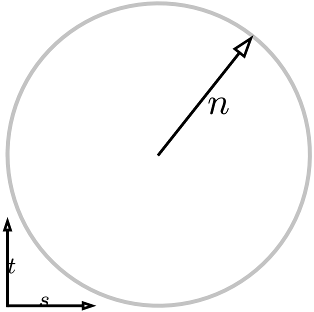

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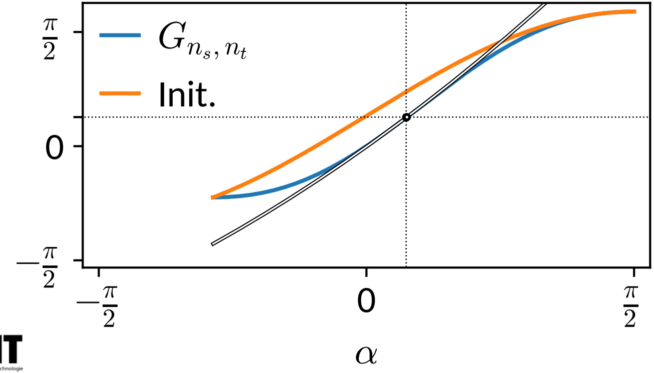
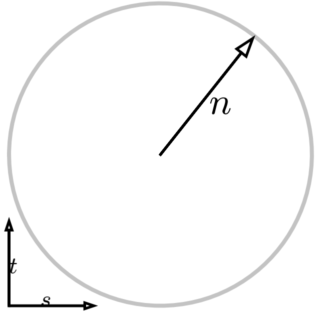




Error analysis

Fix iteration count to 2

Worst case error in ξ: $1.6 \cdot 10^{-5}$

Error analysis

Fix iteration count to 2

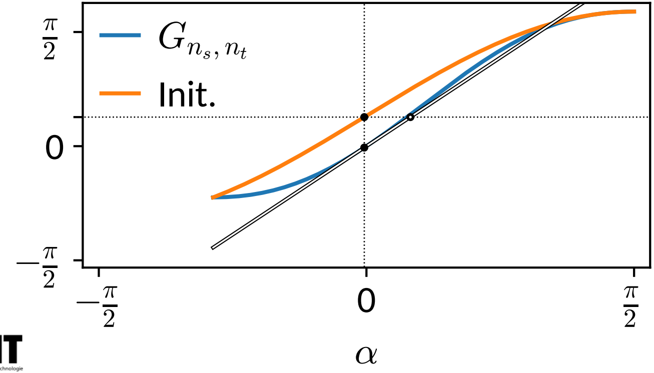
Worst case error in ξ: $1.6 \cdot 10^{-5}$

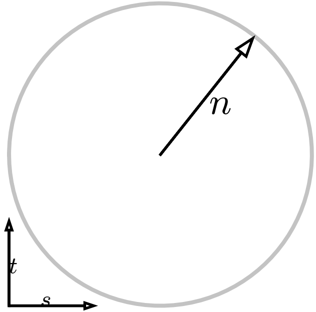
$\rightarrow$ unbiased

Newton's method

$$\alpha := \arcsin \omega_s$$

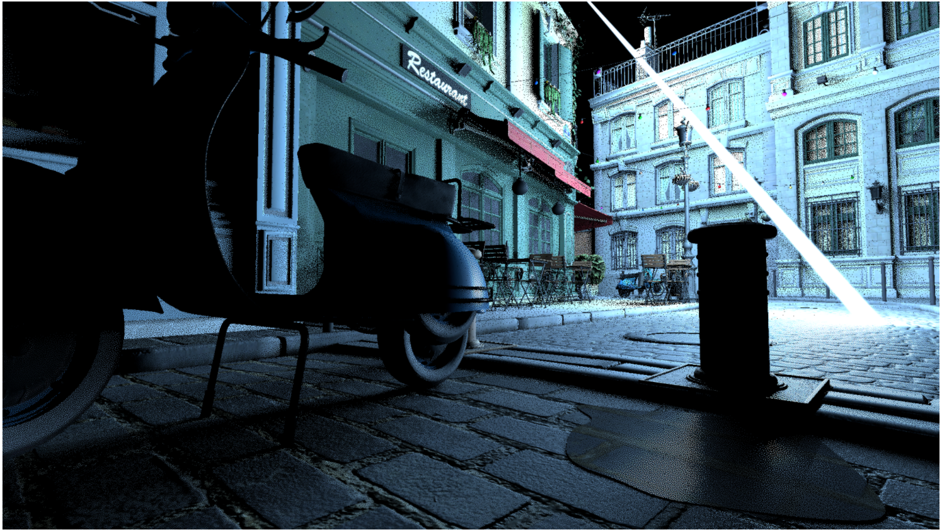
$$G_{n_s,n_t}(\alpha) := F_{n_s,n_t}(\sin\alpha) = n_s \sin^2\alpha + n_t(\cos\alpha \sin\alpha + \alpha)$$





Diffuse and specular shading (1 spp)

Projected solid angle sampling



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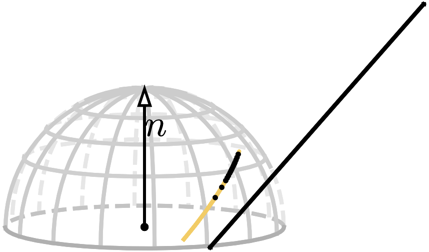
LTC importance sampling



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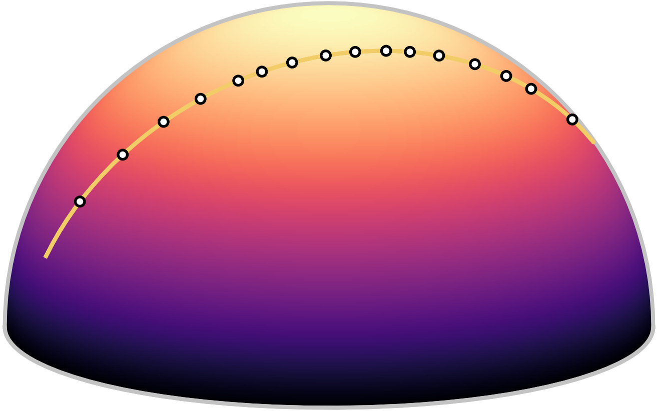
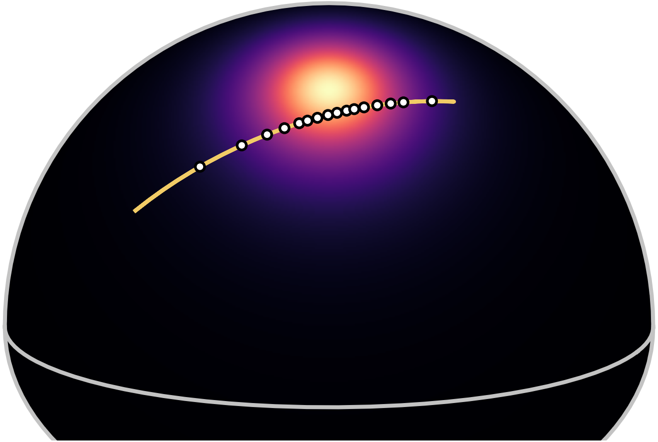
$$p(\omega_i) \propto L_e(\omega_i) f_r(\omega_i, \omega_o)\, n \cdot \omega_i$$

Linearly transformed cosines [Heitz et al. 2016]



cosine distribution

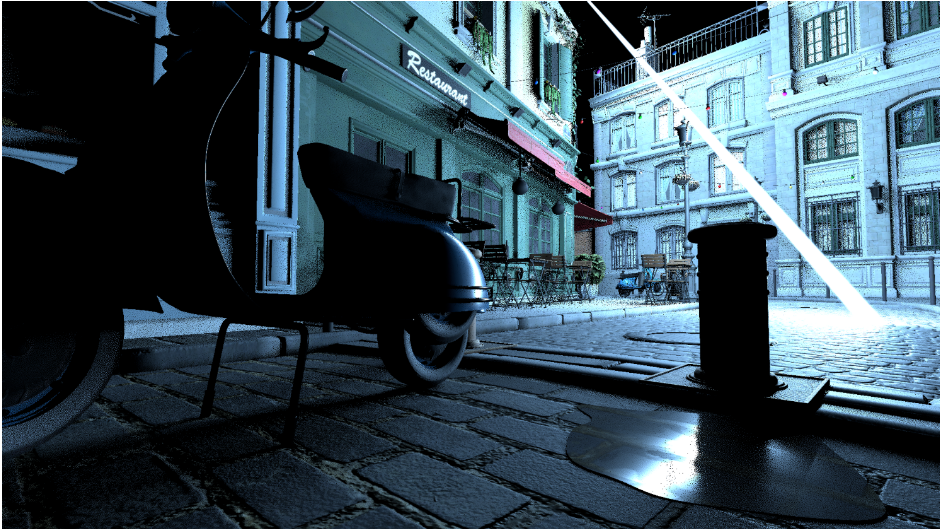
Linearly transformed cosines [Heitz et al. 2016]



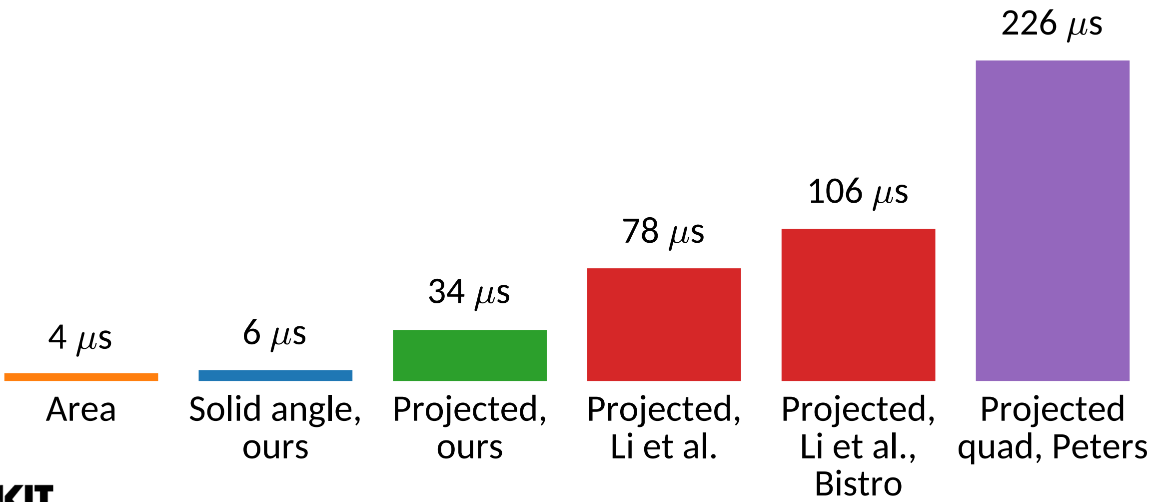
linearly transformed cosine distribution

Diffuse and specular shading (2 spp)

MIS: LTC + projected solid angle sampling



Timings (RTX 2080 Ti, 1920×1080, 1 spp)



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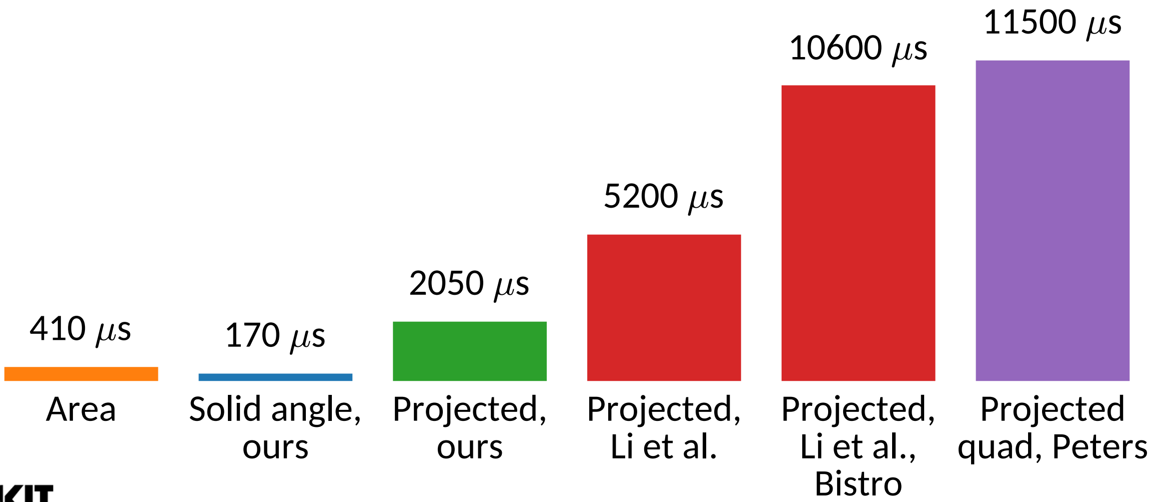
Timings (RTX 2080 Ti, 1920×1080, 128 spp)



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Conclusions

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Put it into your renderer now

Code at https://momentsingraphics.de/HPG2021.html

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Thanks!

Bonus: Stratification

Independent blue noise

$\xi_1 \quad \xi_2 \quad \xi_0$

Jittered uniform blue noise

$\frac{\xi+0}{3} \quad \frac{\xi+1}{3} \quad \frac{\xi+2}{3}$

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