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RESEARCH-ARTICLE

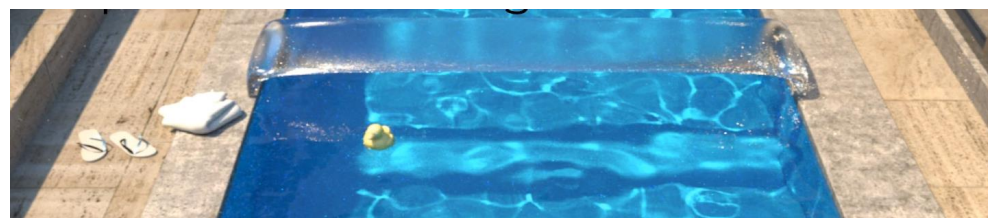
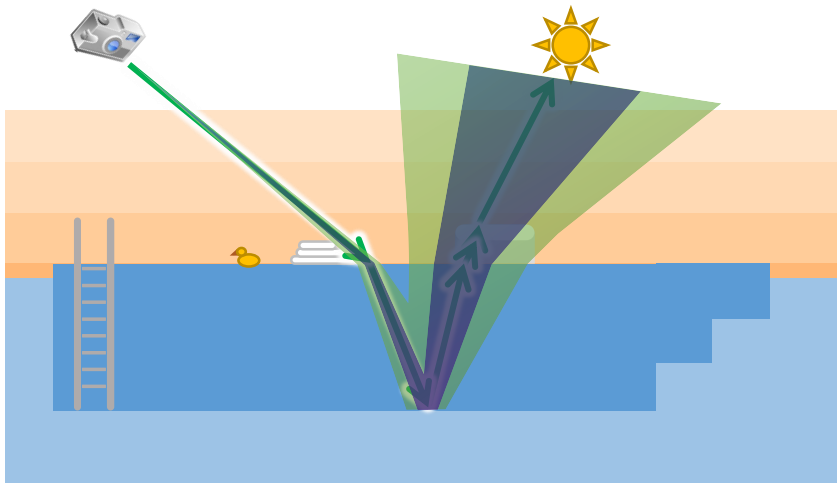
Path differential-informed stratified MCMC and adaptive forward path sampling

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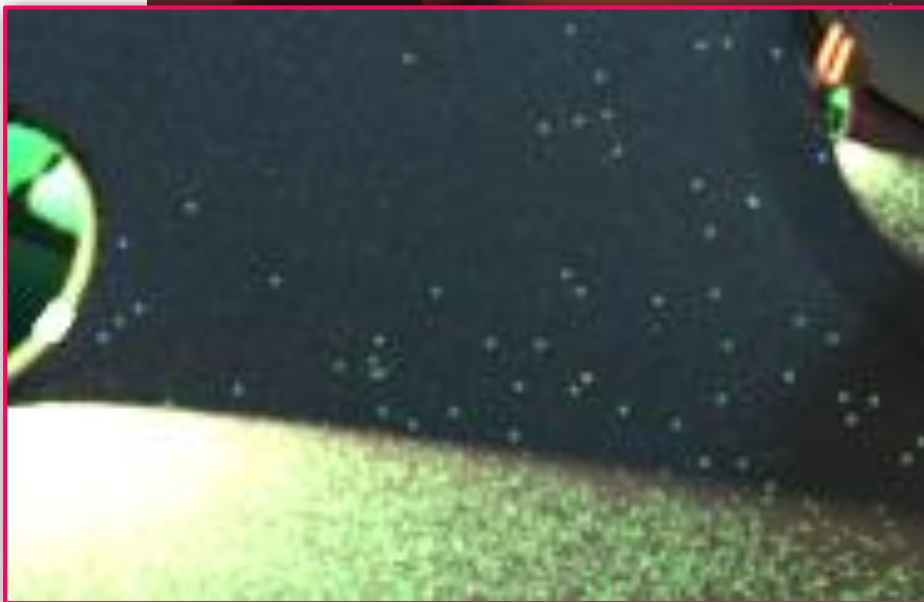
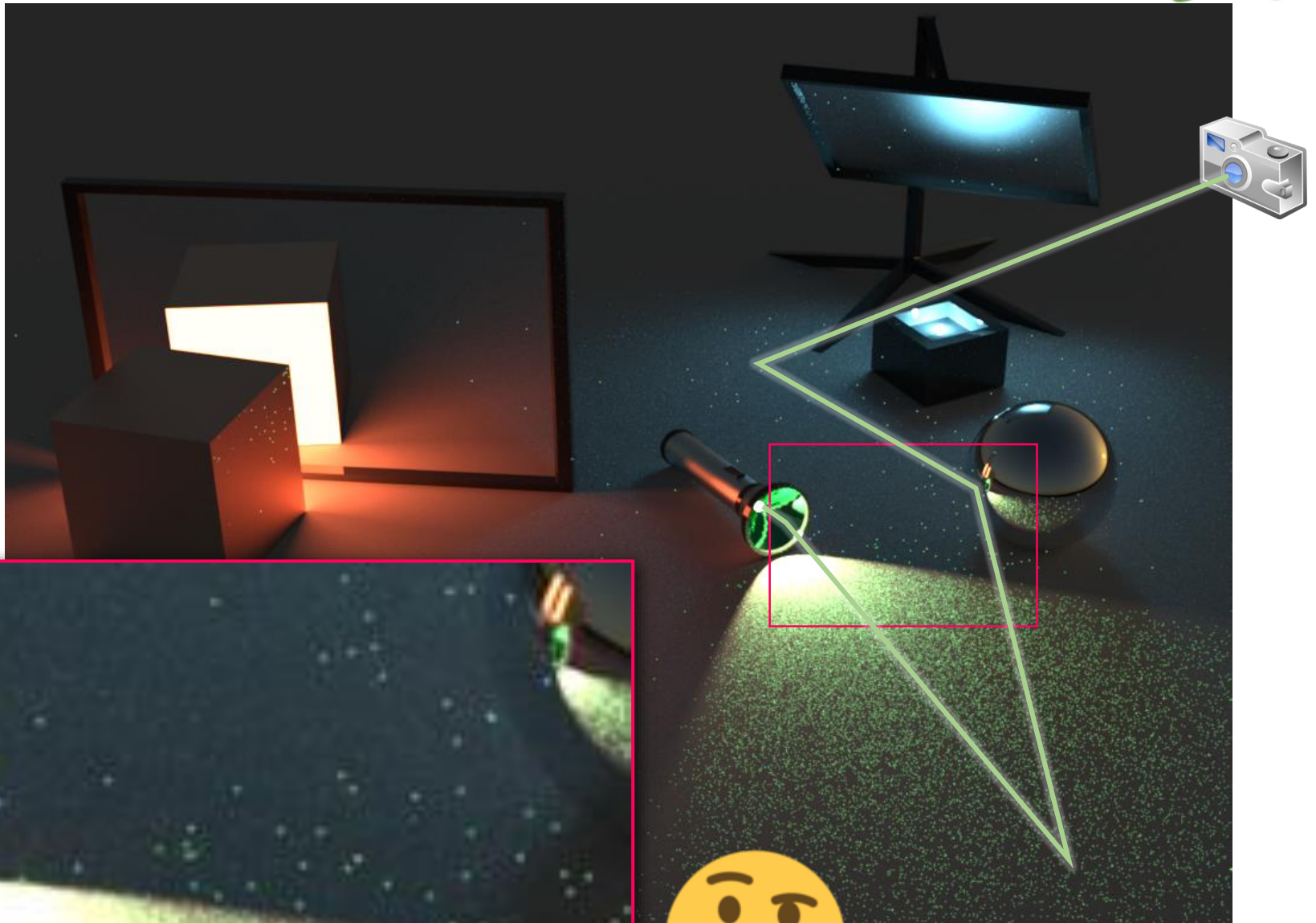
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Why adaptive sampling?

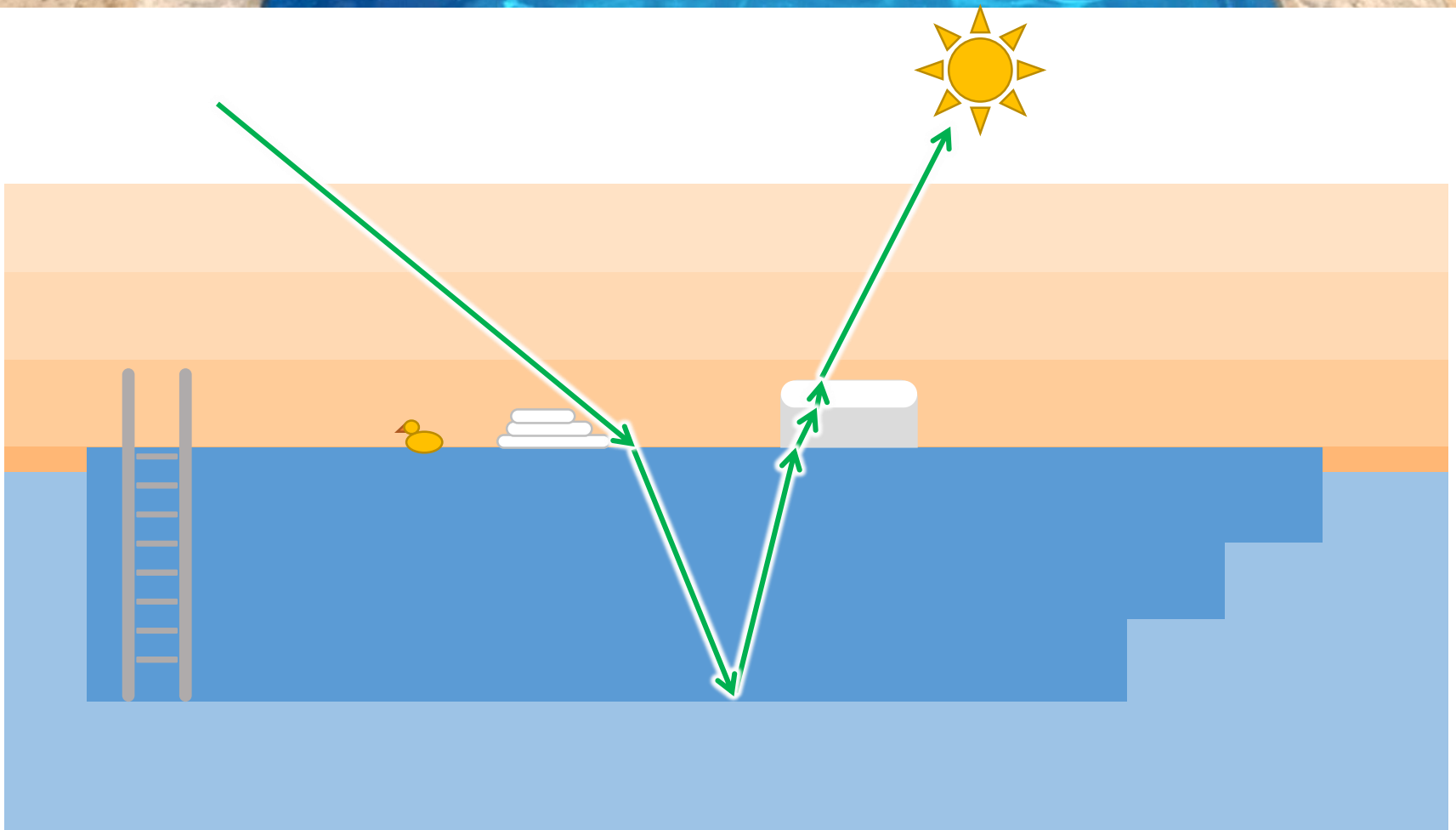
Let's render an image tracing 10.000 random light paths for each pixel!



Not pretty

Some important light paths are rarely found by chance!

Often, those making pretty caustics 🙄

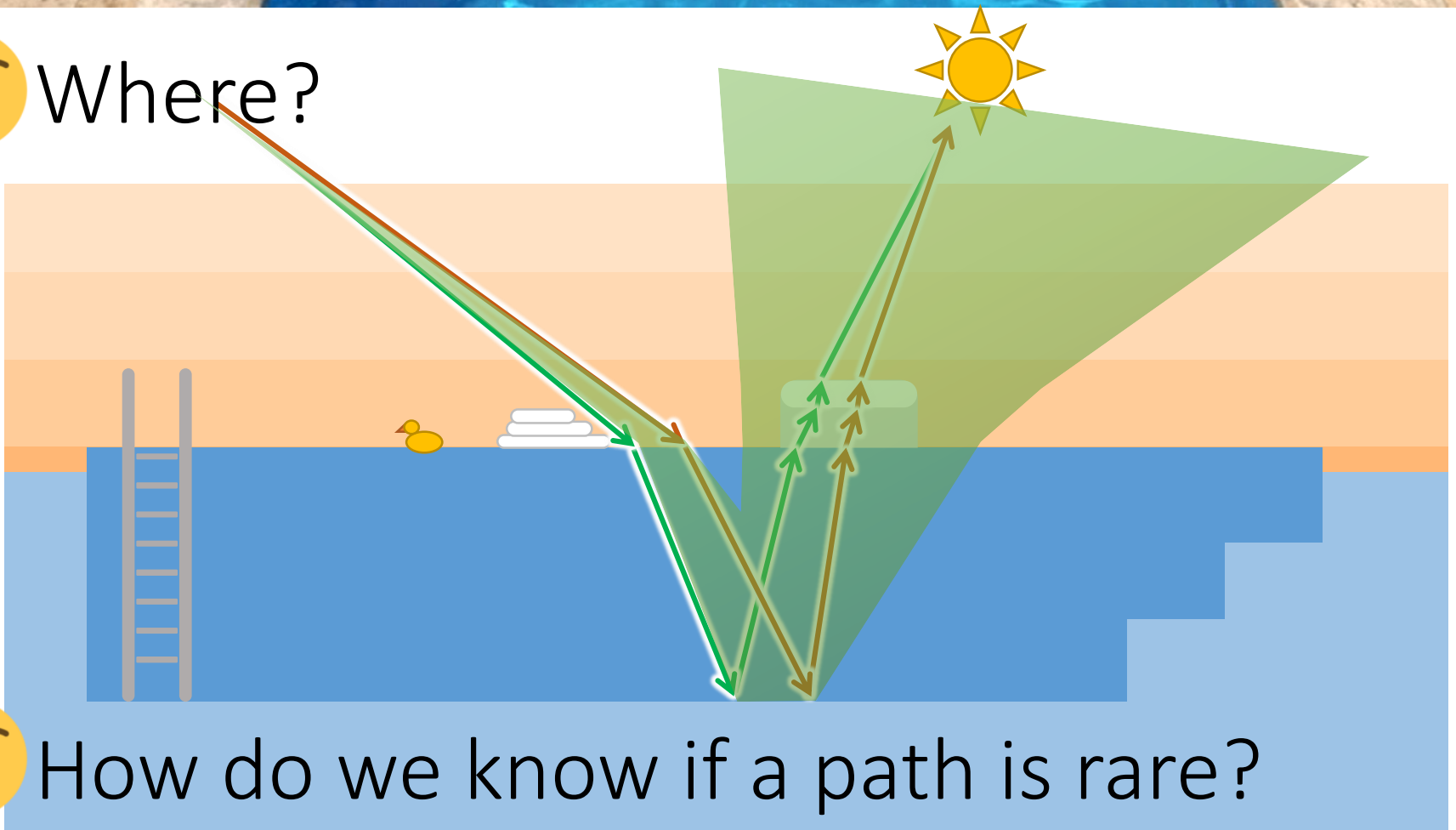


Let's sample adaptively!

When rare light paths are found, we can try to look for more, similar paths



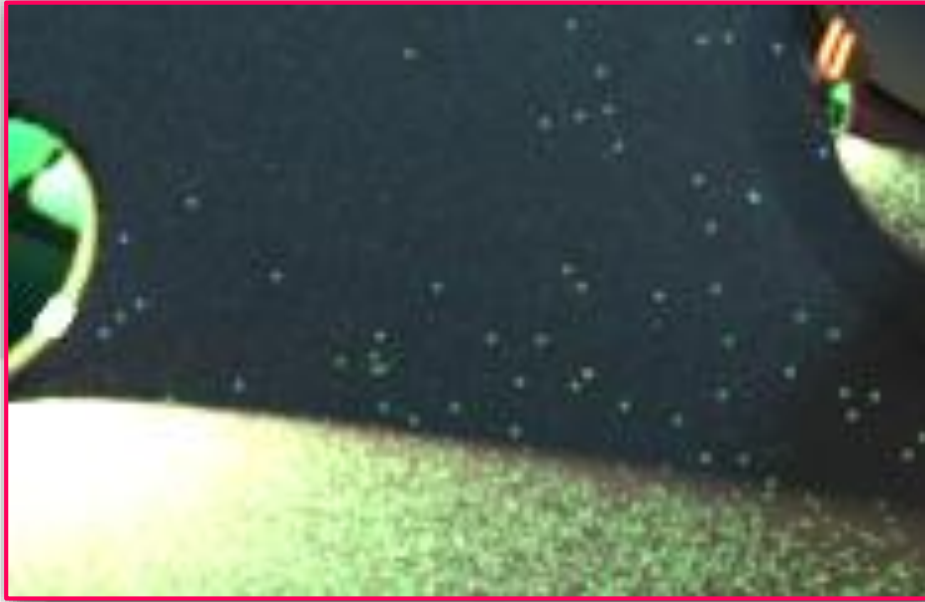
🤔 Where?



🤔 How do we know if a path is rare?

When is a path rare but important?

Its contribution is unusually bright!

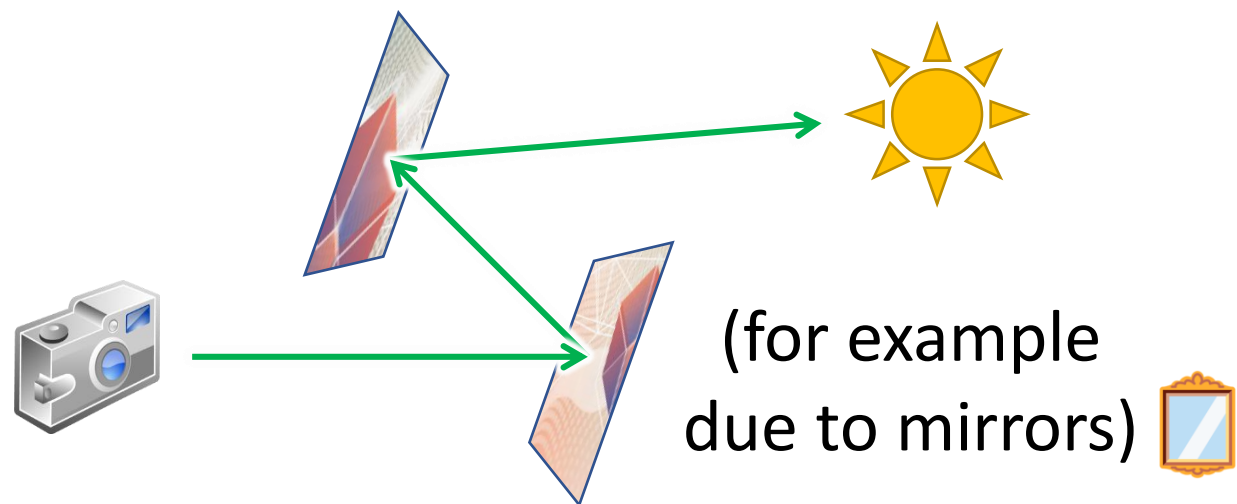


But simple lights are also bright!

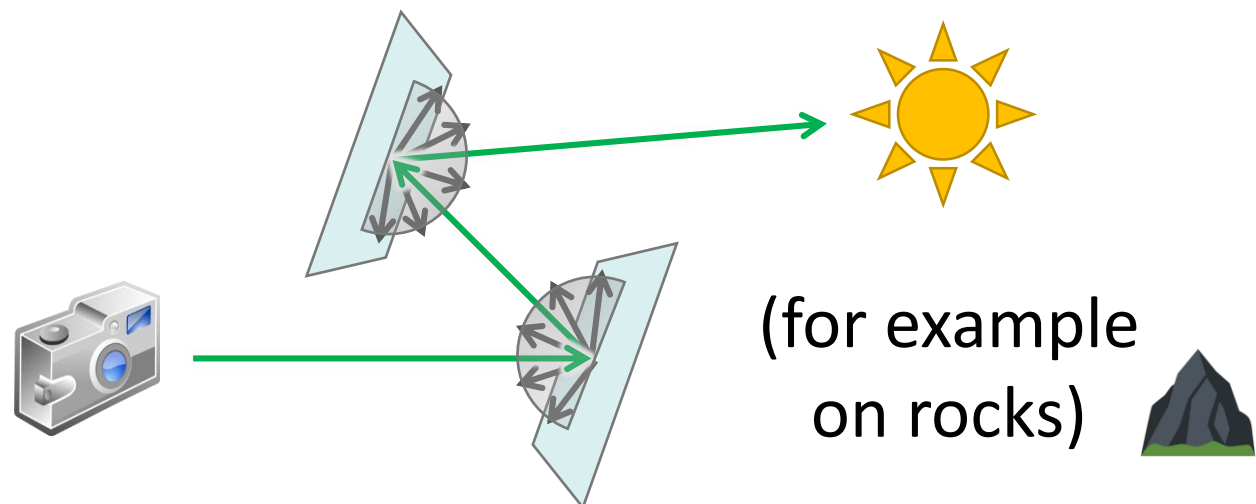


In addition to brightness, let us check how widely paths are scattered! 🧐

Some paths are easy to find, because they all go into the same directions:

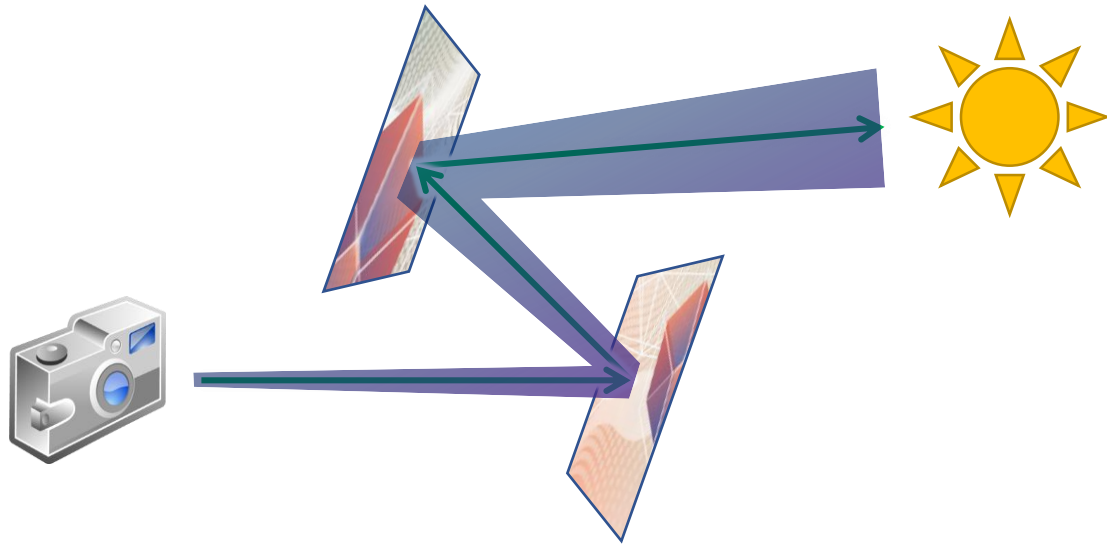


Other paths are scattered into all kinds of directions:

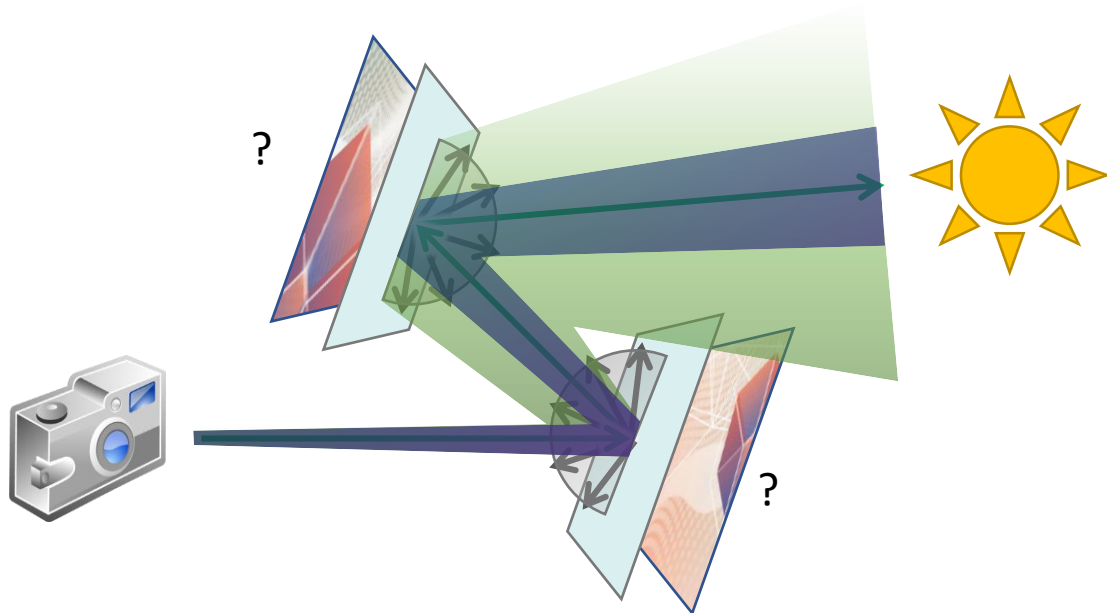



We find that it is enough to know
how a **small set** of paths is scattered 😎

... comparing paths we find easily ...

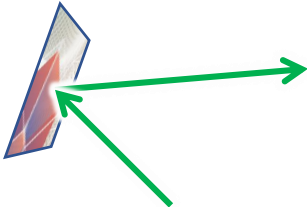


... to paths we find rarely ...



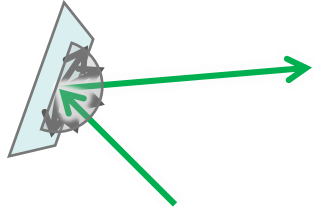
We compute the brightness of paths in a **region** as if they were **easy to find**, compared to the **actual scattering** 

Deterministic transport



$$\int g(U) f^{d|g}(U) dU$$

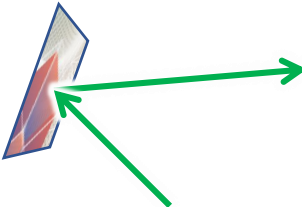
Non-deterministic transport



$$\int g(U) f(U) dU$$

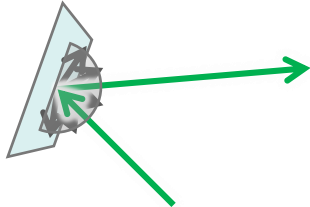
This **ratio** does not get worse when we include paths outside the region!

Deterministic transport



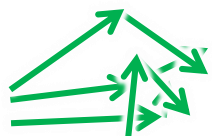
$$\int g(U) f^{d|g}(U) dU$$

Non-deterministic transport




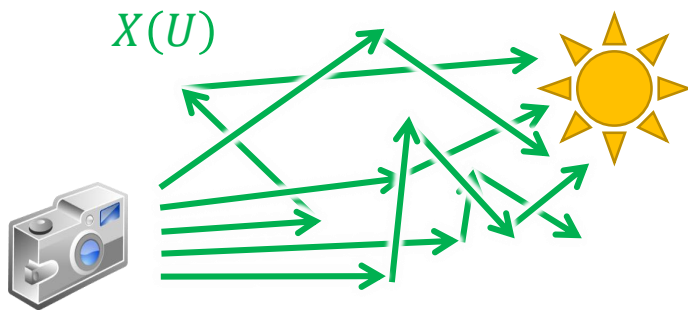
$$\int g(U) f(U) dU$$

Rest of transport



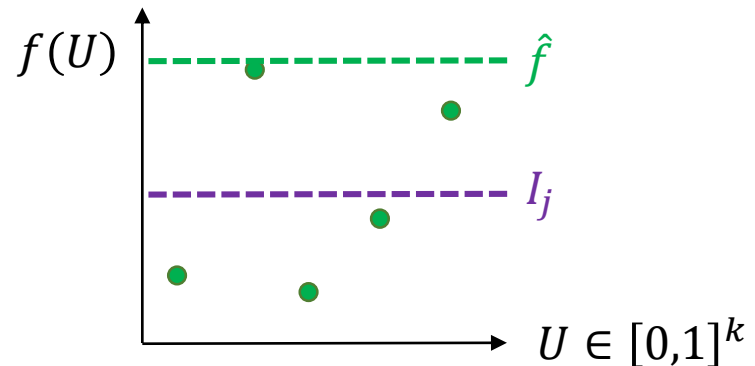
$$\int (1 - g(U)) f(U) dU$$

With our **ratio**, we can bound **potential pixel error** in terms of the differential environment of individual paths! 



Integral:

$$I_j = \int f(U) dU$$



Variance:

$$\mathbf{V}[f(U)] = \int f^2(U) dU$$

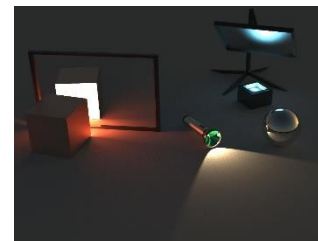
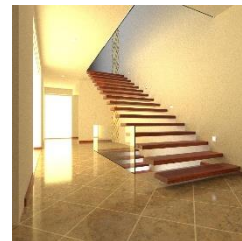
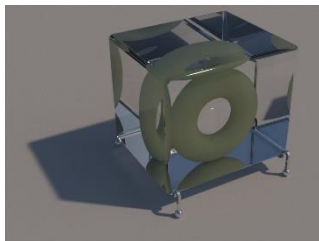
Relative Variance:

$$\frac{\mathbf{V}}{I_j^2} \leq \frac{\hat{f}}{I_j} = r \frac{\hat{f}}{I_j^{d|g}} \leq \tilde{r} \frac{\hat{f}}{I_j^{d|g}}$$

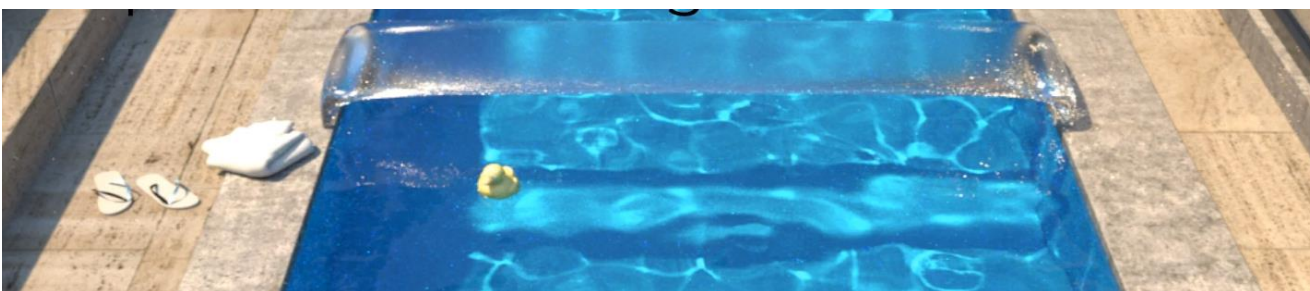
Of course, we cannot anticipate the complexity of entire scenes from one path. The error is bounded **compared to** an unknown, but usually good error of alternative simpler scenarios.

What can we do with it?

Separate good paths from bad paths:

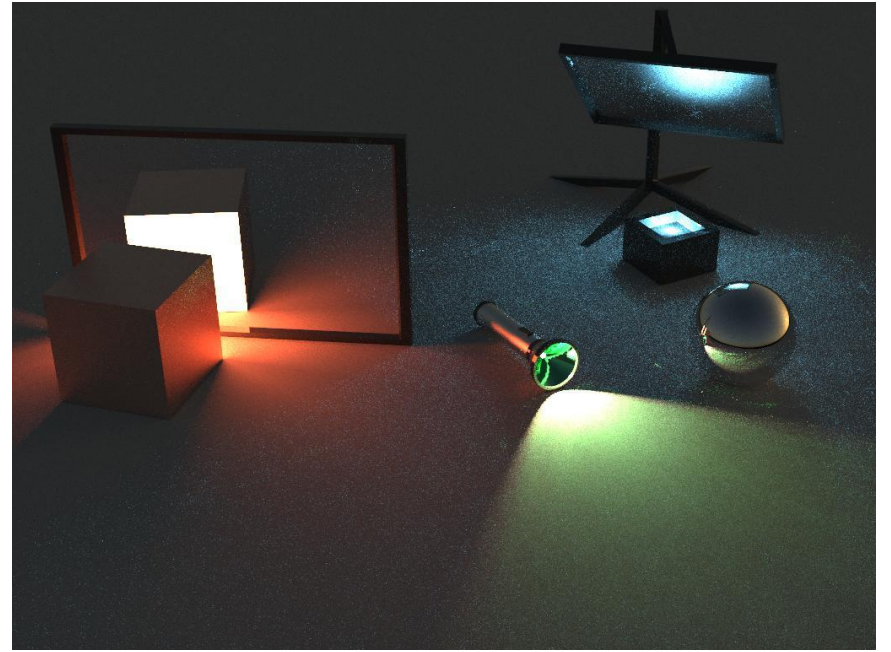
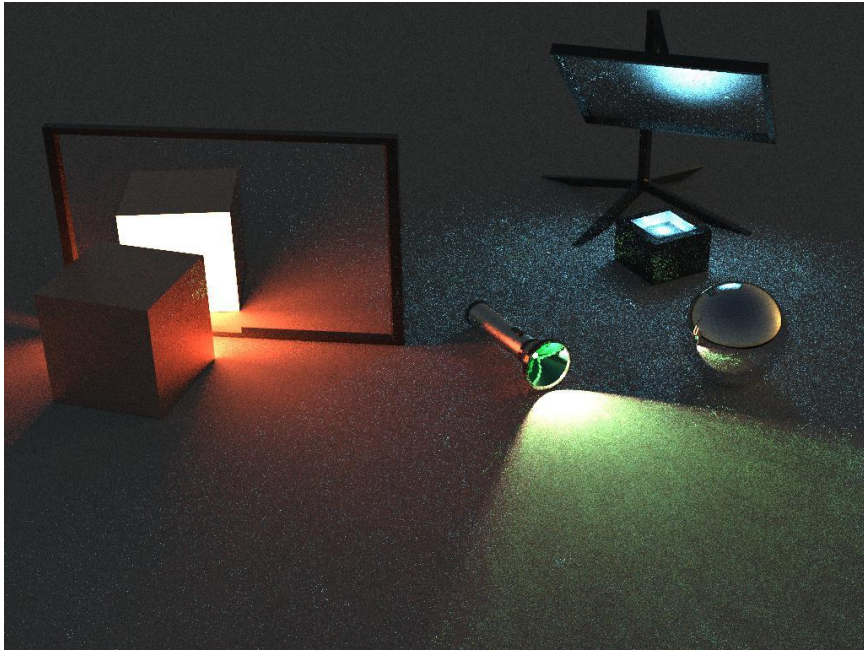


Sample bad paths more adaptively:

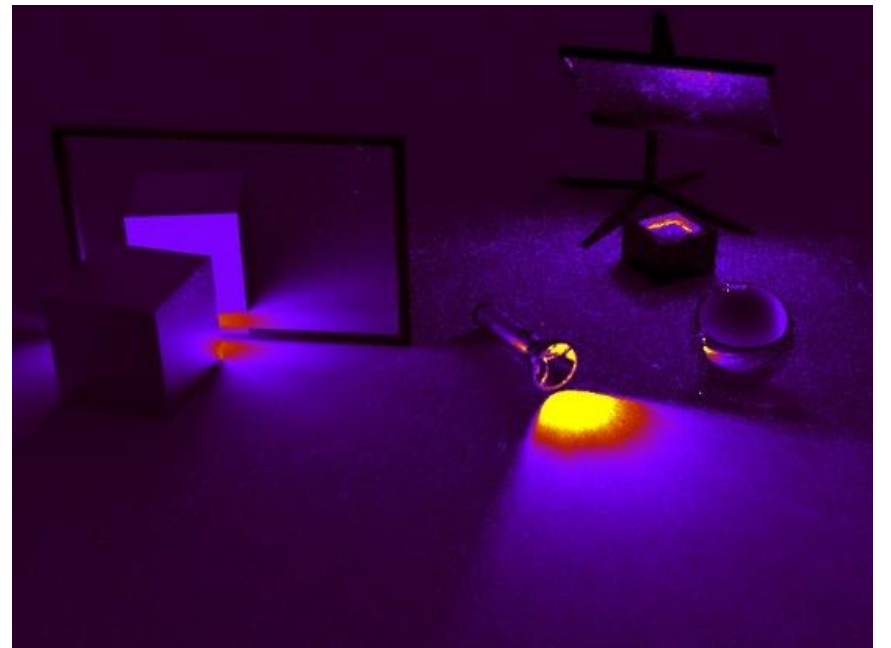
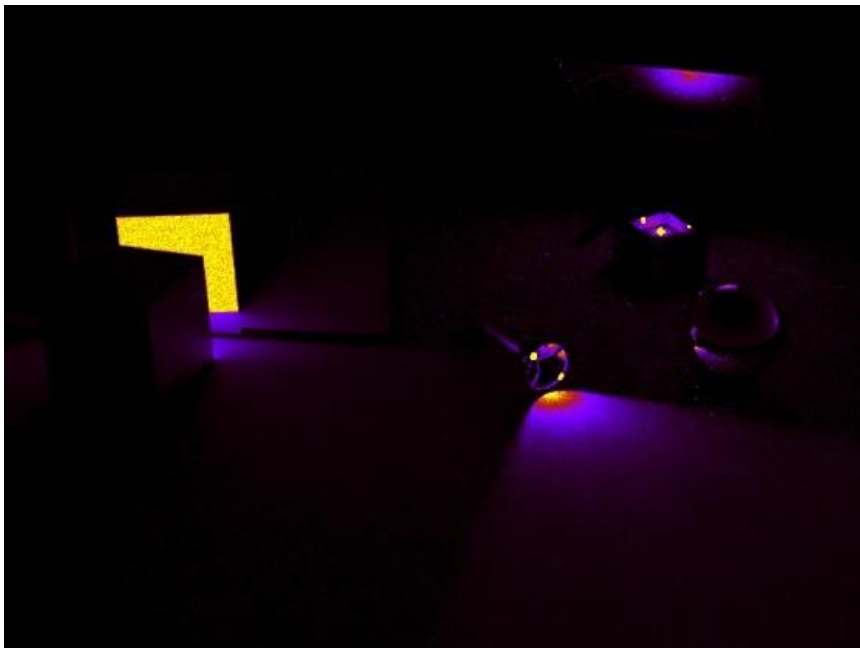


What can we do with it?

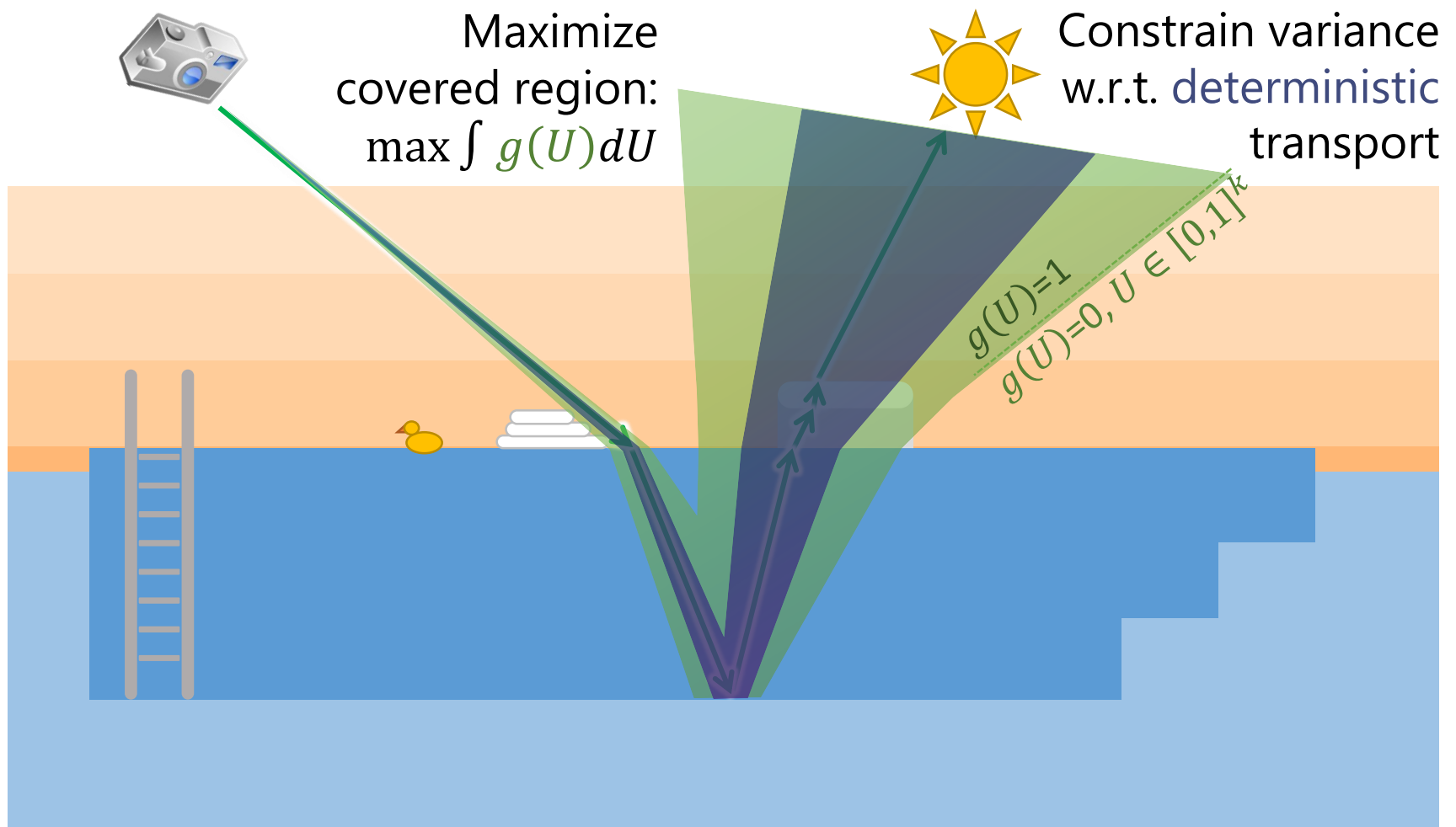
Put samples where we want them ... 🖱️



... i.e. put less samples on easy paths 📊

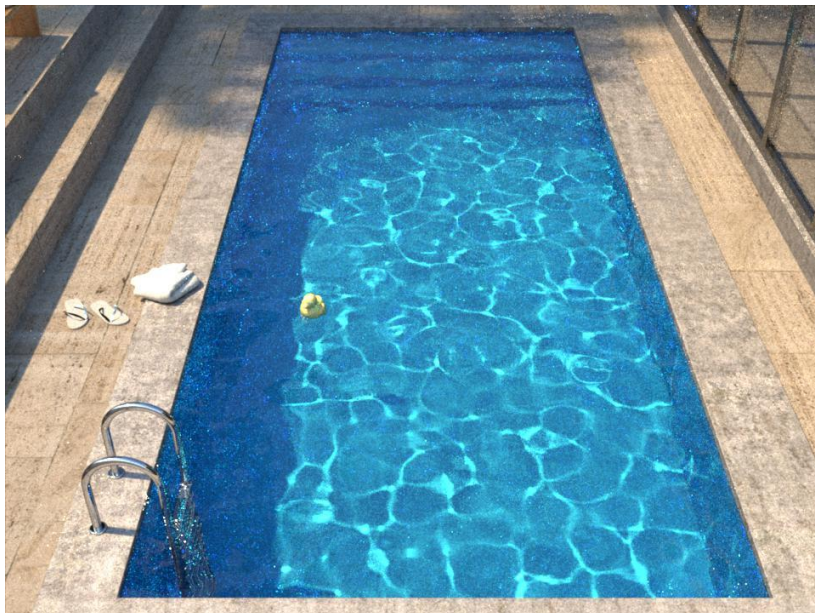
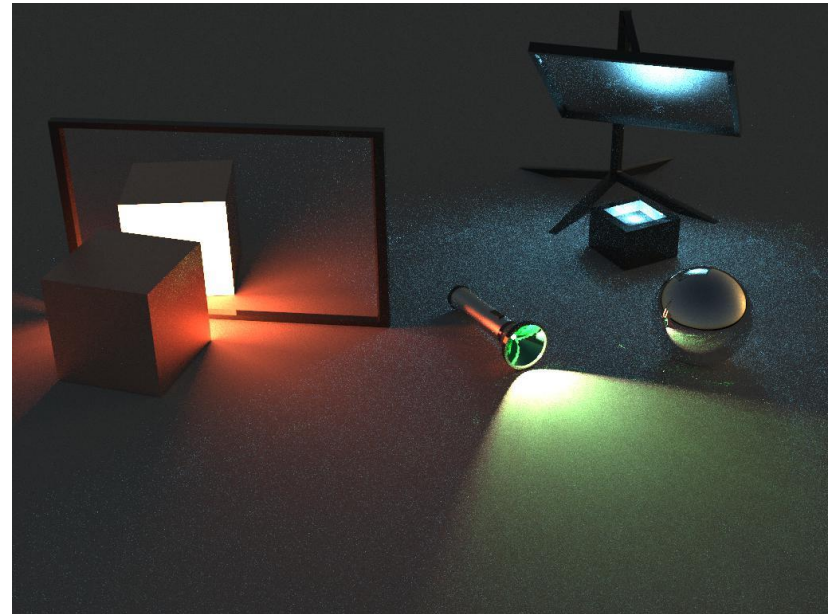
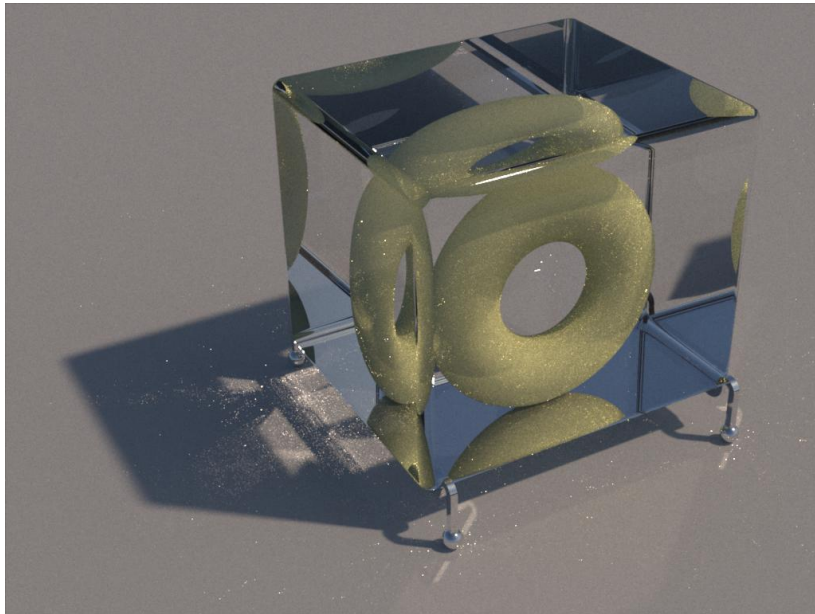


When I find a rare path, **where** do I look for more useful paths?



Using our previously computed subset ratios, that is a simple constrained optimization problem (KKT)! 🎉

Even simple methods, like path tracing from the camera, can give nice results with good adaptive sampling regions 😊



More details in the paper, talk & code! ↗
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