A Simple Diffuse Fluorescent BBRRDF Model

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What is fluorescence?

Visible Light

Wavelength

Energy
Jablonski Energy Diagram

Vibrational Energy Levels

Excited State $S_1$

Ground State $S_0$

Energy States & Levels of Molecules
Absorption $10^{-15}$ s
Relax to $S_1$ $10^{-12}$ s
Fluorescence $10^{-8}$ s
Non-radiative Relaxation
Diffuse
Instantaneous
Emitted wavelength independent of absorbed wavelength, usually longer
Not all absorbed photons get emitted
Not all absorbed photons get emitted:

\[ \Phi = \frac{\text{emitted Photons}}{\text{absorbed Photons}} \]

Wavelength independent
Absorption & Emission Spectrum

Absorption Spectrum

Emission Spectrum

Wavelength (nm)

Relative Intensity (%)
The BBRRDF

Bispectral Bidirectional Reflection and Reradiation Distribution Function (Hullin et al. 2010)

\[
f_r(\omega_{in}, \lambda_{in}, x, \lambda_{out}, \omega_{out}) = \frac{d^2 L_r(x, \omega_{out}, \lambda_{out})}{L_i(x, \omega_{in}, \lambda_{in}) \cos \theta_{in} \ d\omega_{in} d\lambda_{in}}
\]

Wavelengths
The BBRRDF: Previous Work

- Reradiation Matrix
- Discrete description of energy shifts
- Diagonal: non-fluorescent reflectance

Fluorescent red paint Hullin et al. 2010
Our BBRRDF

- Fluorescence is diffuse
  - We use a diffuse BBRRDF
- Fluorescent and non-fluorescent component
  - Light interacts either with fluorescent nor non-fluorescent molecule

Non-fluorescent  
Fluorescent  
Full BBRRDF
Our BBRRDF: Parameters

- $a(\lambda)$ absorption spectrum (scaled to 1)
- $c$ concentration parameter
- $Q$ energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)
Our BBRRDF: Fluorescence

- $a(\lambda)$ absorption spectrum (scaled to 1)
- $c$ concentration parameter
- $Q$ energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

\[
f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = c \cdot a(\lambda_{in}) \cdot Q \cdot e(\lambda_{out}) \cdot \pi^{-1}
\]

Fraction of absorbed energy
Our BBRRDF: Fluorescence

- $a(\lambda)$ absorption spectrum (scaled to 1)
- $c$ concentration parameter
- $Q$ energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = c \cdot a(\lambda_{in}) \cdot Q \cdot e(\lambda_{out}) \cdot \pi^{-1}$$

Fraction of emitted energy
Our BBRRDF: Fluorescence

- $a(\lambda)$ absorption spectrum (scaled to 1)
- $c$ concentration parameter
- $Q$ energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = c \cdot a(\lambda_{in}) \cdot Q \cdot e(\lambda_{out}) \cdot \pi^{-1}$$

Fraction of emitted energy at $\lambda_{out}$
Our BBRRDF: Fluorescence

- $a(\lambda)$ absorption spectrum (scaled to 1)
- $c$ concentration parameter
- $Q$ energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

\[
f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = c \cdot a(\lambda_{in}) \cdot Q \cdot e(\lambda_{out}) \cdot \pi^{-1}
\]

Perfectly diffuse lambert BRDF
Our BBRRDF: Non-fluorescent Reflectance

- $a(\lambda)$ absorption spectrum (scaled to 1)
- $c$ concentration parameter
- $Q$ energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda, \omega_{out}) = (1 - c \cdot a(\lambda)) \cdot r(\lambda) \cdot \pi^{-1}$$

remaining energy
Our BBRRDF: Non-fluorescent Reflectance

- $a(\lambda)$ absorption spectrum (scaled to 1)
- $c$ concentration parameter
- $Q$ energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda, \omega_{out}) = (1 - c \cdot a(\lambda)) \cdot r(\lambda) \cdot \pi^{-1}$$

reflected energy
Our BBRRDF

- $a(\lambda)$ absorption spectrum (scaled to 1)
- $c$ concentration parameter
- $Q$ energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = [\delta_{\lambda_{in}, \lambda_{out}} \cdot (1 - c \cdot a(\lambda_{in})) \cdot r(\lambda_{in})$$

$$+ c \cdot a(\lambda_{in}) \cdot Q \cdot e(\lambda_{out})] \cdot \pi^{-1}$$
Our BBRRDF: Wavelength Sampling

Delta component → 2 Steps:

Sample if light interacts with fluorescent particle

\[ P(\text{fluorescence}) = \frac{\text{fluorescently reflected energy}}{\text{total reflected energy}} \]

If so, sample new wavelength

Camera path: \( p(\lambda_{in}) \propto a \)
Light path: \( p(\lambda_{out}) \propto e \)

Different for camera and light paths!
Energy Conservation vs. Photon Conservation

Energy Conserving BRDF:

\[ \forall \omega_{in} \in \Omega: \int_{\Omega} f(\omega_{in}, \omega_{out}) d\omega_{out} \leq 1 \]
Energy Conservation vs. Photon Conservation

Energy Conserving BBRRDF:

\[
\forall \omega_{in} \in \Omega, \lambda_{in} \in \Lambda: \int_{\Omega \times \Lambda} f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) d(\omega_{out}, \lambda_{out}) \leq 1
\]

Our BBRRDF is energy conserving if

- \(a(\lambda), r(\lambda), Q, c \in [0,1]\)
- \(\int e(\lambda) d\lambda = 1\)

But it is not yet photon conserving!
Energy Conservation vs. Photon Conservation

Example:
- $Q = 1$
- $c = 1$
- Consider $\lambda_{in}$ where $a(\lambda_{in}) = 1$

Energy of a Photon:
$$E = \frac{hc}{\lambda}$$