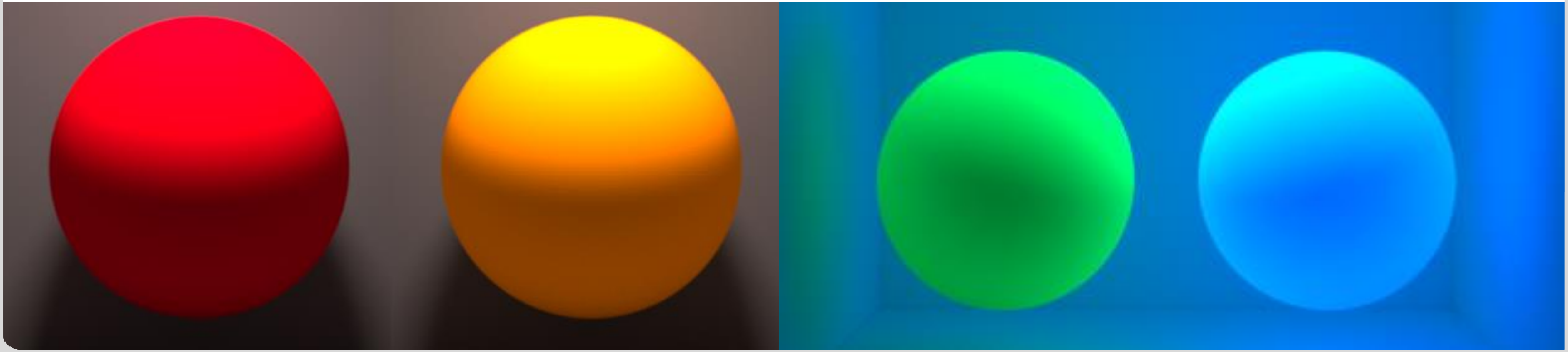
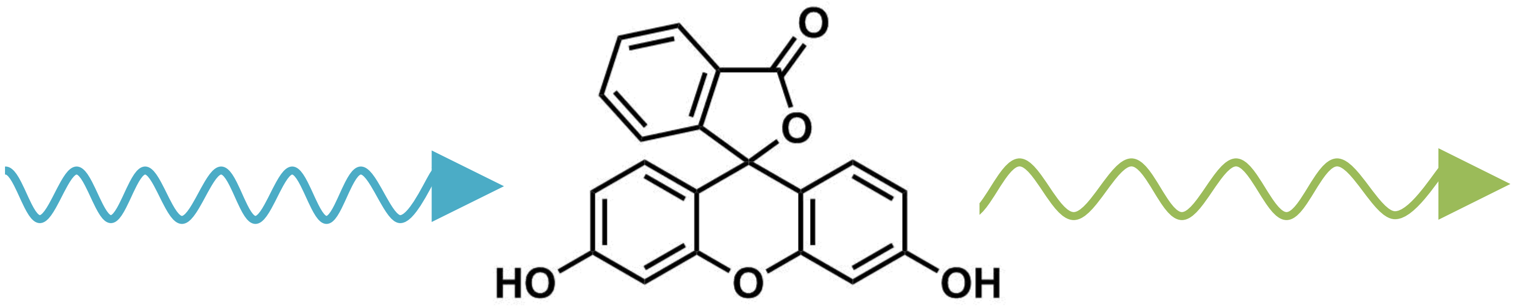
Karlsruher Institut für Technologie

A Simple Diffuse Fluorescent BBRRDF Model

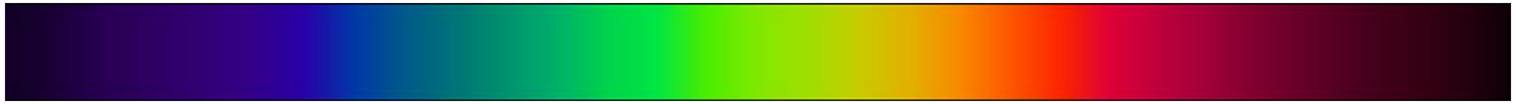
Alisa Jung, Johannes Hanika, Steve Marschner, Carsten Dachsbacher



What is fluorescence?



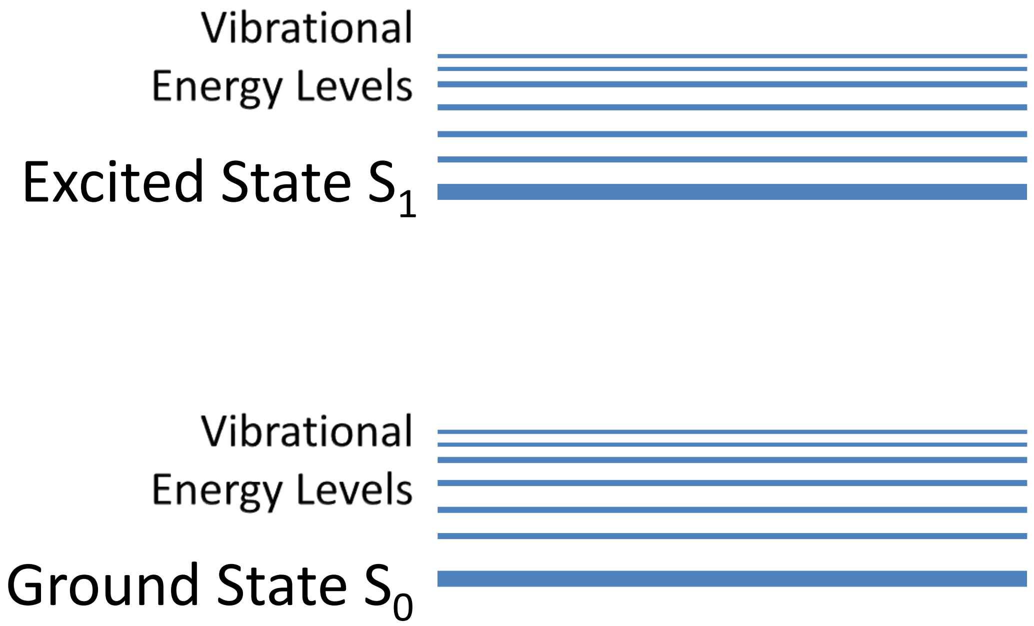
Visible Light



Wavelength

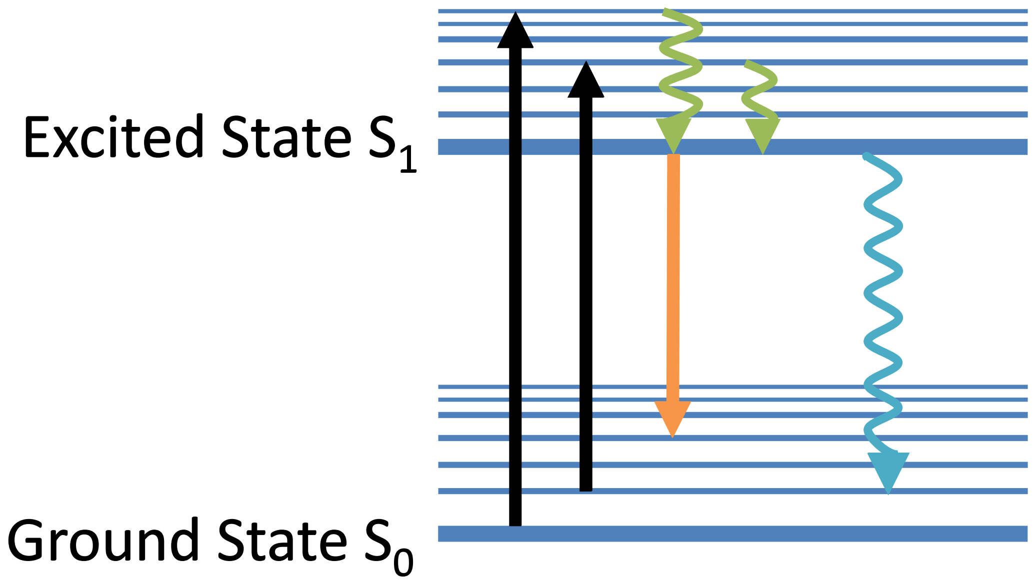
Energy

Jablonski Energy Diagram



- Energy States & Levels of Molecules

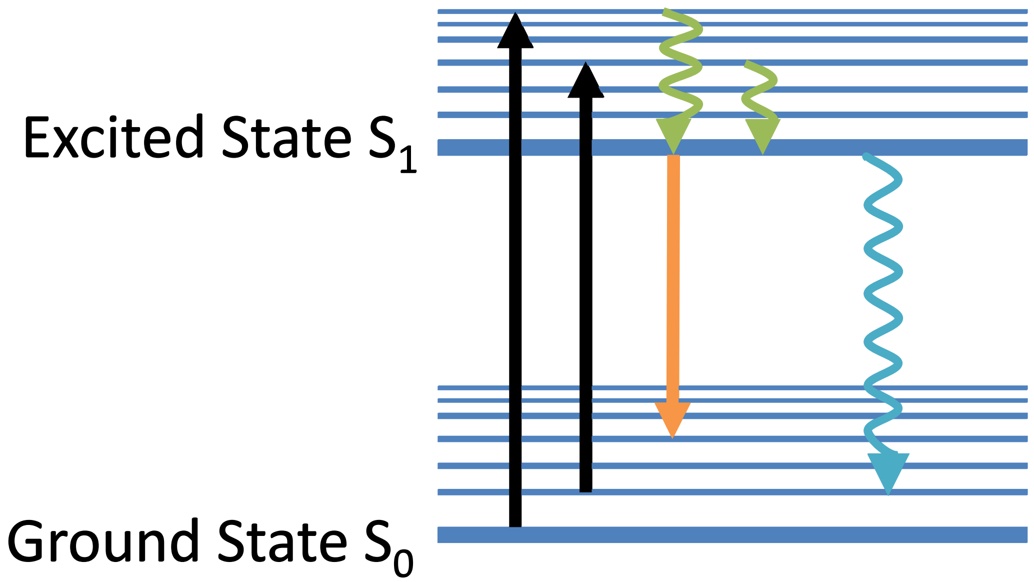
Jablonski Energy Diagram: Fluorescence



- Absorption 10^{-15} s
- Relax to S_1 10^{-12} s
- Fluorescence 10^{-8} s
- Non-radiative Relaxation

- Diffuse
- Instantaneous
- Emitted wavelength independent of absorbed wavelength, usually longer
- Not all absorbed photons get emitted

The Quantum Yield Φ

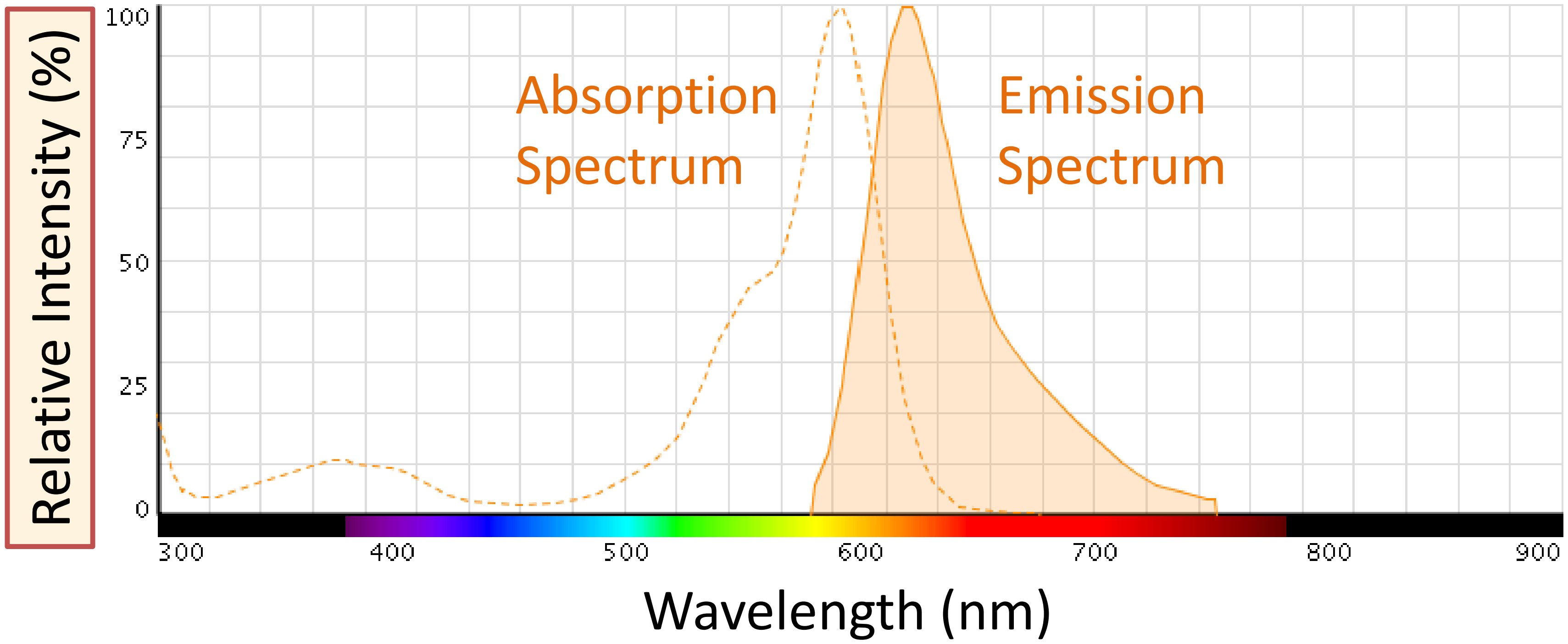


- Not all absorbed photons get emitted:

$$\Phi = \frac{\text{emitted Photons}}{\text{absorbed Photons}}$$

- Wavelength independent

Absorption & Emission Spectrum



The BBRRDF

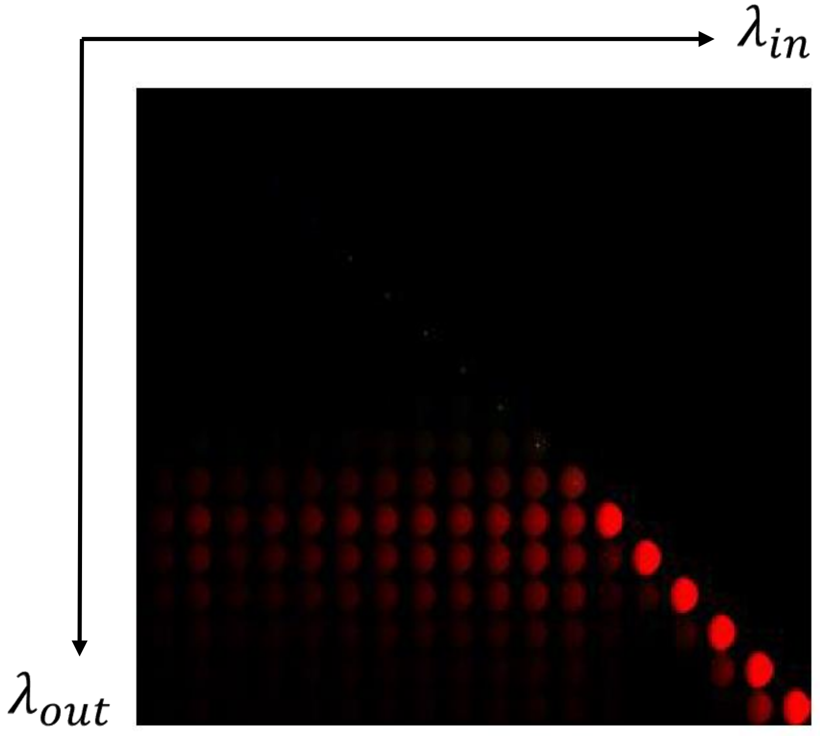
- Bispectral Bidirectional Reflection and Reradiation Distribution Function *(Hullin et al. 2010)*

$$f_r(\omega_{in}, \lambda_{in}, x, \lambda_{out}, \omega_{out}) = \frac{d^2 L_r(x, \omega_{out}, \lambda_{out})}{L_i(x, \omega_{in}, \lambda_{in})\ cos\theta_{in}\ d\omega_{in} d\lambda_{in}}$$

Wavelengths

The BBRRDF: Previous Work

- Reradiation Matrix *(Glassner 1995, Wilkie 2001&2005, Hullin 2010)*
- Discrete description of energy shifts
- Diagonal: non-fluorescent reflectance

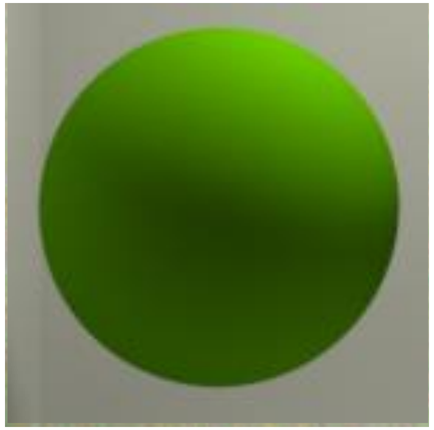


Fluorescent red paint Hullin et al. 2010

Our BBRRDF

- Fluorescence is diffuse
  - We use a diffuse BBRRDF
- Fluorescent and non-fluorescent component
  - Light interacts either with fluorescent nor non-fluorescent molecule


Non-fluorescent

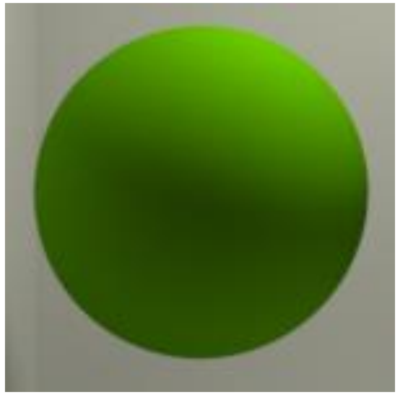

Fluorescent


Full BBRRDF

Our BBRRDF: Parameters

- $a(\lambda)$ absorption spectrum (scaled to 1)
- c concentration parameter
- Q energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

Our BBRRDF: Fluorescence



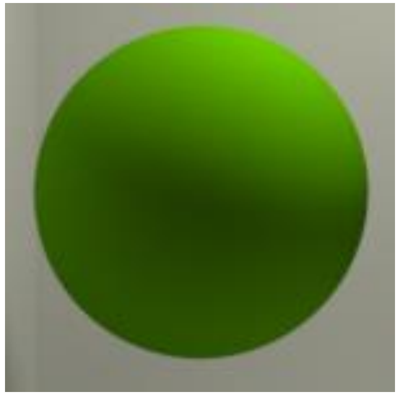
- $a(\lambda)$ absorption spectrum (scaled to 1)
- c concentration parameter
- Q energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = \boxed{c \cdot a(\lambda_{in})} \cdot Q \cdot e(\lambda_{out}) \cdot \pi^{-1}$$

Fraction of absorbed energy

Our BBRRDF: Fluorescence

- $a(\lambda)$ absorption spectrum (scaled to 1)
- c concentration parameter
- Q energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

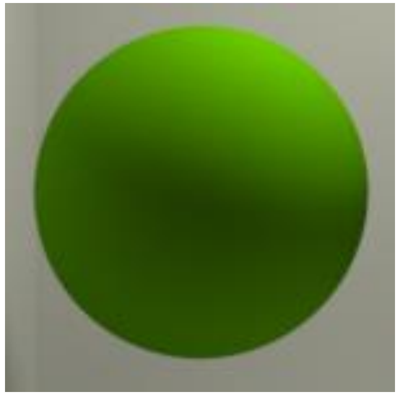


$$f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = c \cdot a(\lambda_{in}) \cdot Q \cdot e(\lambda_{out}) \cdot \pi^{-1}$$

Fraction of emitted energy

Our BBRRDF: Fluorescence

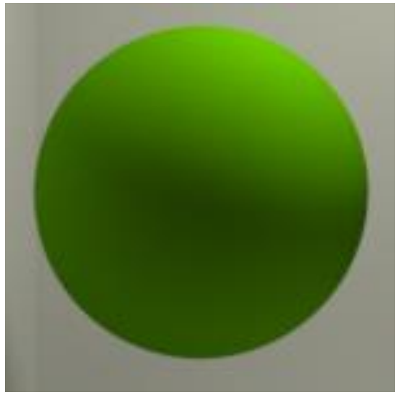
- $a(\lambda)$ absorption spectrum (scaled to 1)
- c concentration parameter
- Q energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)



$$f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = c \cdot a(\lambda_{in}) \cdot Q \cdot e(\lambda_{out}) \cdot \pi^{-1}$$

Fraction of emitted energy at λ_{out}

Our BBRRDF: Fluorescence



- $a(\lambda)$ absorption spectrum (scaled to 1)
- c concentration parameter
- Q energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = c \cdot a(\lambda_{in}) \cdot Q \cdot e(\lambda_{out}) \cdot \pi^{-1}$$

Perfectly diffuse lambert BRDF

Our BBRRDF: Non-fluorescent Reflectance

- $a(\lambda)$ absorption spectrum (scaled to 1)
- c concentration parameter
- Q energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda, \omega_{out}) = \boxed{\left(1 - c \cdot a(\lambda)\right)} \cdot r(\lambda) \cdot \pi^{-1}$$

remaining energy

Our BBRRDF: Non-fluorescent Reflectance

- $a(\lambda)$ absorption spectrum (scaled to 1)
- c concentration parameter
- Q energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda, \omega_{out}) = (1 - c \cdot a(\lambda)) \cdot r(\lambda) \cdot \pi^{-1}$$

reflected energy

Our BBRRDF

- $a(\lambda)$ absorption spectrum (scaled to 1)
- c concentration parameter
- Q energy quantum yield parameter
- $e(\lambda)$ emission spectrum (normed to 1)
- $r(\lambda)$ reflectance spectrum (less than 1)

$$f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) = [\delta_{\lambda_{in}, \lambda_{out}} \cdot \left(1 - c \cdot a(\lambda_{in})\right) \cdot r(\lambda_{in})$$

$$+ c \cdot a(\lambda_{in}) \cdot Q \cdot e(\lambda_{out})] \cdot \pi^{-1}$$

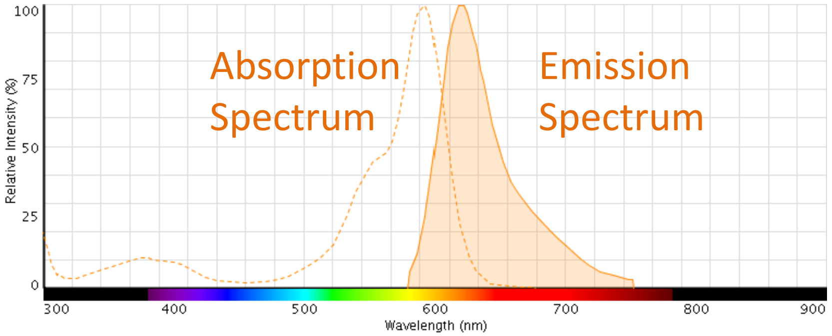
Our BBRRDF: Wavelength Sampling

- Delta component → 2 Steps:
  - Sample if light interacts with fluorescent particle

$$P(\text{fluorescence}) = \frac{\text{fluorescently reflected energy}}{\text{total reflected energy}}$$

  - If so, sample new wavelength

Camera path: $p(\lambda_{in}) \propto a$
Light path: $p(\lambda_{out}) \propto e$



- Different for camera and light paths!

Energy Conservation vs. Photon Conservation

- Energy Conserving BRDF:

$$\forall \omega_{in} \in \Omega\colon \int_{\Omega} f(\omega_{in}, \omega_{out}) d\omega_{out}^{\perp} \leq 1$$

Energy Conservation vs. Photon Conservation

- Energy Conserving BBRRDF:

$$\forall \omega_{in} \in \Omega, \lambda_{in} \in \Lambda: \int_{\Omega \times \Lambda} f(\omega_{in}, \lambda_{in}, \lambda_{out}, \omega_{out}) d(\omega_{out}^{\perp}, \lambda_{out}) \leq 1$$

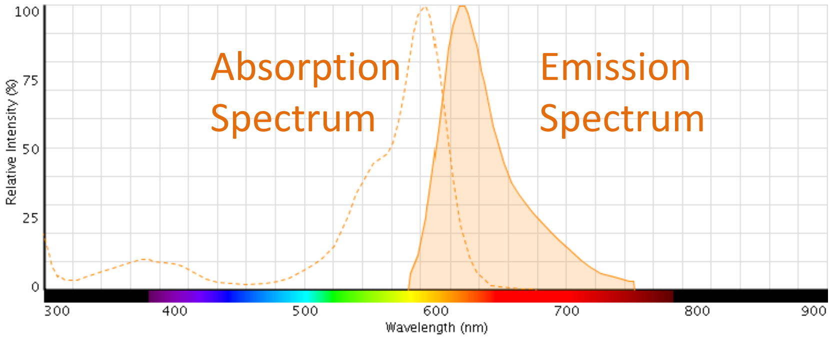
- Our BBRRDF is energy conserving if
  - $\mathrm{a}(\lambda), r(\lambda), Q, c \in [0,1]$
  - $\int e(\lambda) d\lambda = 1$

- But it is not yet photon conserving!

Energy Conservation vs. Photon Conservation

Example:

- $Q = 1$
- $c = 1$



- Consider λ_{in} where $a(\lambda_{in}) = 1$

Energy of a Photon:

$$E = \frac{hc}{\lambda}$$