Interactive Light Scattering with Principal-Ordinate Propagation

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ABSTRACT

Efficient light transport simulation in participating media is challenging in general, but especially if the medium is heterogeneous and exhibits significant multiple anisotropic scattering. We present a novel finite-element method that achieves interactive rendering speeds on modern GPUs without imposing any significant restrictions on the rendered participated medium. We achieve this by dynamically decomposing all illumination into directional and point light sources, and propagating the light from these virtual sources in independent discrete propagation volumes. These are individually aligned with approximate principal directions of light propagation from the respective light sources. Such decomposition allows us to use a very simple and computationally efficient unimodal basis for representing the propagated radiance, instead of using a general basis such as Spherical Harmonics. The presented approach is biased but physically plausible, and largely reduces rendering artifacts inherent to standard finite-element methods while allowing for virtually arbitrary scattering anisotropy and other properties of the simulated medium, without requiring any precomputation.

Index Terms: I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Radiosity; I.6.8 [Simulation and Modeling]: Types of Simulation—Parallel

1 INTRODUCTION

Scattering, or translucency, greatly contributes to the appearance of many natural substances and objects in our surrounding. Albeit the problem can be easily formulated as the radiance transfer equation [3, 21], computing a solution can be very costly. Consequently, many existing approaches simplify the problem, e.g., by assuming isotropic scattering or homogeneity of the material, to achieve interactive performance.

In this work we propose a novel interactive algorithm for plausible rendering of heterogeneous participating media with arbitrary anisotropy. The core of our approach is to propagate light in propagation volumes oriented along the principal ordinates of the source illumination. For this we typically use multiple rectilinear grids to propagate environmental (distant) lighting, and spherical grids to account for point light sources. In both cases, one dimension of the grids is aligned with the prominent directional part of the source radiance for which the grid has been created. In contrast to previous methods (e.g., [1, 13]), discretizing the illumination into directional and point light sources enables us to approximately describe the anisotropy (directionality) of light transport by a single scalar value per grid cell. Specifically, this anisotropy value corresponds to a unimodal function implicitly aligned with the respective principal ordinate. In addition to exploiting data locality and the parallelism of GPUs, the benefit of these decisions is a significant reduction of the false scattering and ray effect artifacts arising in many finite-element methods as a consequence of representing the propagated radiance by, e.g., spherical harmonics or piecewise-constant functions.

Our main contributions can be summarized as follows:

- We introduce a novel approach to finite-element light propagation using implicitly aligned unimodal distributions for regular and spherical grids. This helps reducing propagation artifacts and helps to preserve the directionality of light during the propagation.
- A simplification of lighting by decomposing both environmental and surface illumination (via virtual point lights) into separate principal ordinate grids.
- An observer-centric importance-based selection of principal ordinates and prefiltering for environment lighting, helping to hide its discretized character used in the propagation.

2 PREVIOUS WORK

Offline methods A range of different approaches has been presented to compute solutions to the radiance transport equation for participating environments [3, 21]. However, none of the classic techniques provides a satisfying combination of generality, robustness, and, most importantly in our context, speed. Unbiased Monte-Carlo methods, such as bidirectional path tracing [18] and Metropolis light transport [26] usually require a large number of paths to be traced; in particular in dense media with high scattering anisotropy and albedo (like clouds or milk) the computation time increases tremendously. Caching is often used to speed up the computation, e.g., radiance caching [10], photon mapping [11, 12] or virtual point lights [6].
However, these methods typically do not handle highly anisotropic scattering very well, even with recent improvements [24, 25], and their performance is often far from interactive.

**Finite-Element methods** Finite-element methods, including volume radiosity [29], the discrete ordinates method (DOM) [3], light diffusion [32], and lattice-Boltzmann transport (LB) [8] handle highly multiple scattering well. However, in practice they allow only isotropic or moderately anisotropic scattering, and usually suffer from false scattering (smoothing of sharp light beams) and ray effects (selective exaggeration of scattered light due to discretized directions). Light propagation maps [7] significantly reduce the artifacts, but are still limited to rather moderate anisotropy. It can therefore be seen that strong scattering anisotropy is one of the main limiting factors for existing methods. This is unfortunate, as most real-world media exhibit relatively high anisotropy (Henyey-Greenstein [9] coefficient \( g \approx 0.9 \) or more [23]). Although isotropic approximations are acceptable in some cases, this is generally not a valid assumption and one of the primary motivations for our work.

**Interactive rendering** Numerous works focus on individual optical phenomena to achieve interactive or real-time performance. These phenomena include light shafts [5, 28], volume caustics [17, 19], shadows [20, 30], and clouds [2]. Various approaches can also be found in classic visualization literature, e.g., half-angle slicing [15] which empirically compiles forward scattering for volume visualization. Sometimes precomputation is used to speed up the rendering of heterogeneous translucent objects [31, 33] or smoke using compensated ray marching [34]. In contrast, we target general multiple scattering in participating media without any precomputation or focus on a particular phenomenon.

We build on concepts of DOMs and light propagation volumes [1, 13]. These approaches are attractive for interactive applications as their grid-based local propagation schemes allow for easy parallel implementation on contemporary GPUs. Virtually all existing variants of DOM use a single scene-aligned grid, where every grid cell stores a representation of the directional radiance function using spherical harmonics (SH) or piecewise-constant functions. This representation is then used to iteratively calculate energy transfer between nearby cells, typically within a local 18- or 26-neighbourhood. However, this representation is only suited for moderately anisotropic scattering at best; especially for anisotropic media under complex (high-frequency) illumination such approach causes prominent ray effect and false scattering artifacts (see [7]). We take a different approach and propose to identify the most important light propagation directions (principal ordinates) in the scene and then use multiple propagation grids aligned with these directions, instead of a single volume. This enables using a unimodal representation of the angular energy distribution around the principal direction in each grid cell.

### 3 Principal Ordinate Propagation

The core idea of our method is to reduce the main drawbacks of previous grid-based iterative methods, false scattering and ray effects, by using propagation volumes where the propagation domain is explicitly aligned with approximate principal directions of light transport. Furthermore, we use only a single scalar value per grid cell to describe the local anisotropy of the directional light distribution. In our scheme, we use the well-known Heney-Greenstein (HG) [9] distribution; the aforementioned value, called the anisotropy coefficient, is used to parametrize this distribution. Using principal directions implies that for more complex lighting scenarios we have to use multiple grids that sufficiently well approximate their directionality; for local light sources we propose to use spherical grids centred around them.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>( d )</td>
<td>(principal) direction</td>
</tr>
<tr>
<td>( g )</td>
<td>scattering anisotropy coefficient</td>
</tr>
<tr>
<td>( \sigma_s, \sigma_a )</td>
<td>scattering / absorption coefficient</td>
</tr>
<tr>
<td>( x_i )</td>
<td>location of grid cell ( i )</td>
</tr>
<tr>
<td>( L_i, n_i )</td>
<td>(per-cell) radiance magnitude and anisotropy</td>
</tr>
<tr>
<td>( f_{\text{hg}}, F_{\text{hg}} )</td>
<td>HG function and its cumulative distribution</td>
</tr>
<tr>
<td>( \mu )</td>
<td>scattering angle cosine</td>
</tr>
<tr>
<td>( L, L_{\text{acc}} )</td>
<td>propagation and accumulation grid</td>
</tr>
<tr>
<td>( M, n )</td>
<td>number of iterations / iteration index</td>
</tr>
<tr>
<td>( I_i(d) )</td>
<td>incident radiance from direction ( d )</td>
</tr>
<tr>
<td>( \Delta I_{\text{refl-acq}} )</td>
<td>src to dst radiance contribution</td>
</tr>
<tr>
<td>( T_i, T_{\text{refl-acq}} )</td>
<td>transmittance to cell ( i ) and between cells</td>
</tr>
<tr>
<td>( \Omega_s, \Omega_a )</td>
<td>solid angle subtended by cell ( i ) or ordinate ( n )</td>
</tr>
<tr>
<td>( N, n )</td>
<td>number of principal ordinates / ordinate index</td>
</tr>
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Table 1: Table of symbols (in the order of appearance).

These choices assume that the principal directions can be derived from the initial radiance distribution and do not change strongly when light travels through the medium. However, such variation might occur if the density of the simulated medium changes abruptly. Still, as we discuss in Sec. 3.1.5, violating this assumption does not cause our algorithm to fail, but only leads to decreasing its accuracy.

In the following we first detail our concept of principal ordinate propagation for a single directional source (Sec. 3.1). Then we describe how to extend this scheme to environment illumination (Sec. 3.2) and local light sources (Sec. 3.3) by using multiple importance-sampled rectilinear and spherical propagation volumes, respectively. The propagation scheme is explained using radiance as the radiometric quantity; we assume all other quantities (such as irradiance from environment maps or intensity from point lights) to be converted accordingly. All frequently-used notation is summarized in Table 1.

#### 3.1 Regular grids for directional light

The concept as well as the propagation scheme can be best explained for parallel (distant) light travelling along a direction \( d \) through a region in space (Fig. 2). For this case we discretize the space into a uniform rectilinear grid similar to DOM; however, we make sure that one of its dimensions is aligned with \( d \). For every grid cell \( i \), we store the directional distribution of light and its magnitude \( L_i \) (all computations are performed independently per-wavelength, which is omitted here for brevity). The main difference to DOM is that we represent both the directional distribution of light and the phase function using the HG distribution implicitly aligned with \( d \). To distinguish radiance anisotropy (directional distributions) from phase functions, we denote the HG parameter for the former as \( a_i \in [-1, 1] \), and \( g \in [0, 1] \) for the latter (we do not consider negative values of \( g \) because of physical implausibility of dominantly-backscattering
media). That is, the directional radianc of a grid cell centred at \( \mathbf{x}_i \) is 
\[ L(\mathbf{x}_i, \omega) = L_i \cdot f_{hg}(\mu, \alpha_i) \], where \( f_{hg} \) is the HG function and \( \mu = \omega \cdot \mathbf{d} \)
\( \text{is the cosine of the angle between a direction } \omega \text{ and the principal light direction } \mathbf{d} \). We assume that the medium is further characterized by its (spatially-varying) scattering coefficient \( \sigma_s \) and absorption coefficient \( \sigma_a \); these two quantities as well as the spatially-varying anisotropy of the phase function defined by the HG parameter \( g \) are wavelength-dependent and stored for every cell of the medium volume (which exists independently of the propagation volumes).

Conceptually, two grids are required in the propagation procedure. The first, *propagation grid*, stores the unpropagated (residual) energy; we will denote it as \( L \) and its state at the iteration \( m \in \{1..M\} \), where \( M \) is the total number of propagation iterations, as \( L^m \). The second, *accumulation grid* \( L_{acc} \), is needed to accumulate the energy transported through the medium over the course of the computation.

Two options are available for implementing \( L_{acc} \): we could either store the overall propagation distribution that has passed through each cell during the propagation, or alternatively store only the observer-dependent out-scattered radiation at each iteration. We opted for the second approach, because storing the entire directional radiation distribution at each cell is much more expensive than just accumulating the outgoing radianace (which is essentially a single scalar value). Although this course requires recomputing the solution on every observer position change, it is in agreement with our premise of a fully dynamic algorithm without relying on precomputations.

### 3.1.1 Grid initialization

At the beginning each propagation grid—which is scaled to span the entire medium (Fig. 2, top)—needs to be initialized by the incident radianace at each cell. As no scattering has been accounted for yet, the anisotropy is set to an HG coefficient of \( \alpha_i = 1 \), an equivalent to the Dirac function in the direction \( \mathbf{d} \) (Fig. 3). The radianace magnitude \( L_i \) is set to the incident radianace \( L_{in}(\mathbf{d}) \) at \( \mathbf{x}_i \), attenuated by absorption and out-scattering. That is, for every cell, we compute the transmittance \( T_T \) (from the point where light enters the medium, travelling along \( \mathbf{d} \) to \( \mathbf{x}_i \) set to \( L_i = L_{in}(\mathbf{d}) \cdot T_T \). Note that this can be efficiently computed using ray marching: as our grid is aligned with \( \mathbf{d} \) we can compute the transmittance incrementally along individual ‘slices’ of the grid along \( \mathbf{d} \) in a single pass.

### 3.1.2 Light energy propagation

In this section, we describe how to iteratively update the grid to simulate the propagation of light. We use a propagation stencil where the radianace of each grid cell is propagated to its 6 direct neighbours in every iteration. More specifically, we perform a gathering-type computation of how much radianace flows into each grid cell from its neighbours based on their radianace distributions and then combine these contributions to yield the new distribution at that cell (Fig. 4, right). In the following we denote the neighbouring source cell with index src, and the target destination cell with dst.

**Radiance magnitude contribution** We first need to determine the amount of radiant energy that flows from cell src towards dst according to the radianace distribution in src. To this end, we efficiently compute the integral of \( L(\mathbf{x}_{src}, \omega) \) over the solid angle subtended by dst (denoted as \( \Omega_{src\rightarrow dst} \) below) using the closed form of the cumulative HG function \( F_{hg}(\mu, g) = \int_0^1 f_{hg}((1+g^2-2g\mu)^{1/2} - \frac{1}{1+g}) \) \( d\mu \):

\[
F_{hg}(\mu, g) = \frac{1 - g^2}{4\pi g} \left( \frac{1}{(1 + g^2 - 2g\mu)^{1/2}} - \frac{1}{1 + g} \right).
\]

By this we compute the radianace from src travelling towards dst using the transmittance \( T_{src\rightarrow dst} \) as

\[
\Delta L_{src\rightarrow dst} = L_{src} \cdot T_{src\rightarrow dst} \cdot |\phi_1 - \phi_2| \cdot (F_{hg}(\cos \phi_1, a_{src}) - F_{hg}(\cos \phi_2, a_{src}))
\]

using the following approximate parametrization for the subtended solid angle \( \Omega_{src\rightarrow dst} \) (depending on mutual positions of src and dst):

\[
(\theta_1, \theta_2, |\phi_1 - \phi_2|) = \begin{cases} 
(0, \frac{\pi}{2}, 2\pi) & \text{dst in front of src} \\
(\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}) & \text{dst next to src} \\
(\frac{3\pi}{2}, \pi, 2\pi) & \text{dst behind src}
\end{cases}
\]

(see Fig. 4, left for a sample illustration of the second case of Eq. 3). Since the HG distribution is rotationally-symmetric (Fig. 4, middle) only the absolute value of the difference of the azimuthal angles \( |\phi_1 - \phi_2| \) is required. Note that here the transmittance \( T_{src\rightarrow dst} \) accounts just for absorption that affects the radianace propagation on its way from src to dst. This is because our scheme treats scattering as a decrease of anisotropy and not as an extinction process, as we show below. In practice, we take the averaged absorption coefficients \( \sigma_s \) at the source and destination cells and the distance between their centres \( d \), and apply the Beer-Lambert-Bouguer law; however, ray-marching with a small number of steps might potentially be required to integrate the absorption coefficient if the resolution of the propagation volume is much smaller than the medium grid.

**Radiance anisotropy contribution** Similarly to absorption attenuating the radiant energy flowing between neighbouring cells, the anisotropy of the energy propagated from src to dst will decrease due to scattering. In agreement with the radiance transfer equation, in our case this can be easily computed exploiting the self-convolution property of the HG distribution [22]: in a medium with scattering anisotropy of \( g \) the radiance anisotropy reduces to \( a' = a \cdot g^{a/4} \) after...

![Figure 3: The propagation grid aligned with the direction of incidence is initialized with the attenuated radiances and an anisotropy parameter \( a_i = 1 \). During the propagation both radiance magnitude and anisotropy change towards lower anisotropy.](image-url)

![Figure 4: Left: Our polar parametrization of the solid sphere. The coloured patches correspond to the approximate solid angles subtended by the cells next to (green), in front (purple) and behind (orange) src. Middle: The HG cumulative function \( f_{hg} \) is used to integrate the radiances from the source cell flowing towards the destination cells (depicted as coloured patches of \( f_{hg} \), for \( g = 0.5 \)). Right: On the way the light undergoes scattering and is possibly reduced by absorption.](image-url)
travelling a distance \( t \) (assuming a constant \( \sigma_a \) along this path). We obtain \( \sigma_a \) and \( t \) the same way as for computing \( T_{src\rightarrow dst} \) above. The change of radiance anisotropy from \( src \) to \( dst \) is therefore

\[
\Delta L_{src\rightarrow dst} = a_{src} \cdot g^{\Delta t}.
\]

(4)

We can easily see that this formula cannot lead to an increase of anisotropy, since \( g \in [0,1] \). Additionally, in non-scattering media (\( \sigma_a = 0 \)) the anisotropy will be preserved perfectly.

Combining contributions from neighbours

Updating the radiance distribution at the cell \( dst \) entails accumulating the contributions from its six neighbours (indexed by \( src \)) as

\[
L_{dst} = \sum_{src} \Delta L_{src\rightarrow dst},
\]

(5)

\[
a_{dst} = \frac{\sum_{src} \Delta L_{src\rightarrow dst} \cdot \Delta a_{src\rightarrow dst}}{\sum_{src} \Delta L_{src\rightarrow dst}}.
\]

(6)

While the radiant energy contributions simply need to be added up, the anisotropy is a weighted average of its neighbours, since the update has to yield an anisotropy value \( a_{dst} \) within the valid range. We discuss implications of Eq. 6 in Sec. 3.1.5.

3.1.3 Iterating the solution

The update procedure defined by Eqs. 5 and 6 is performed for every cell of \( L \) to yield \( L \) for every iteration \( m \). Implementation-wise, this requires maintaining a second grid identical to the propagation grid and swapping these at each iteration.

Additionally, the results of every propagation iteration need to be accumulated in \( L_{acc} \) by evaluating the updated distributions in \( L^{m+1} \):

\[
L_{acc,i}^{m+1} = L_{acc,i}^m + L^{m+1}(x_i, c - x_i)
\]

(7)

\[
= L_{acc,i}^m + L^{m+1}_i \cdot f_{hg}(\mu, a_i^{m+1})
\]

(8)

for every cell \( i \). Here \( c \) is the observer position and \( \mu \) is therefore the dot product of \( d \) and the view direction.

3.1.4 Upsampling and rendering

When the solution has converged after a sufficient number of iterations, using it for rendering is relatively straightforward. We employ ray-marching to integrate the incoming radiance for every camera ray using the common front-to-back emission-absorption model [21]. In this case the emission term corresponds to the scattered radiance accumulated in \( L_{acc} \).

As we discuss in Sec. 4, the typical resolutions used for the propagation grids need to be rather small (in most of our examples \( 20^3 \) or less) for performance reasons. In order to improve the rendering quality with such low grid resolutions it is desired to upsample them prior to their visualization. We use a 3D version of the joint bilateral upsampling [16] where the density field of the medium (i.e., the spatially varying scattering coefficient) is used as a guidance signal. Typically, the density field is significantly more detailed than the propagation volumes; this detail is "transferred" to the solution by the upsampling. According to our experiments, low-resolution propagation grids are usually sufficient for plausible results.

3.1.5 Discussion of the propagation scheme

Using the unimodal HG function with a single parameter to represent the directional distributions in light transport obviously means that there are distributions in a cell that cannot be represented well. On the other hand, we compensate for this by using multiple grids (see Sec. 3.2), which in turn can handle anisotropic phase functions significantly better than previous work thanks to the proposed propagation scheme. In comparison, an exceedingly large number of SH coefficients is required to represent highly anisotropic distributions, and this still does not prevent false scattering issues if a local propagation scheme is employed.

In this view the most heuristic step of our scheme is the recombination of reduced anisotropies from the neighbouring cells in Eq. 6. The logic behind this formulation is that the radiance distribution at \( dst \) will result from superposing the neighbouring distributions according to how much energy they contribute to \( dst \). The main limitation of this approach lies in the fact that combining multiple HG distributions with different anisotropy values cannot generally be represented by any single HG distribution. Although we have experimented with fitting the resulting HG distribution to the combination of its neighbours in terms of least square error, we found that the simple weighted arithmetic average produces comparable results while keeping the computational cost of this core operation minimal. In addition, Eq. 6 very well preserves the anisotropy of light transported along the principal direction, thus greatly reducing false scattering effects.

Note that there are cases of very heterogeneous media where our approach might locally become too inaccurate (see Fig. 5). If light along the principal direction undergoes strong absorption, while light from other directions does not, the resulting light distribution should possibly become skewed, which cannot be represented within our framework. Although this is obviously a failure case of our representation, occurrences of such strong absorption fluctuations are comparatively rare, and more importantly the resulting radiance magnitude in these cases is typically very small (therefore having little impact on the resulting image). Also note that with multiple propagation volumes we can actually reproduce complex multimodal radiance distributions, despite each grid being composed of unimodal HG distributions.

3.2 Environment lighting: Multiple propagation grids

In the previous section we have described our approach for a single directional light source. In order to account for environmental lighting (typically modelled by an environment map), we need to use multiple grids oriented in different principal directions. In the following we discuss how to choose these directions and, as every grid accounts for light from a finite solid angle, how to prefilter the respective incident radiance to avoid singularity artifacts (see Fig. 6).

Prefiltering

A straightforward approach is importance-sampling the environment map to obtain \( N \) directions, \( d_n \), each carrying an
Importance map

Ours (g = 0) Ours (g = 0.7) Ours (g = 0.81) Ours (g = 0.8)

Light tracing (g = 0.7) Light tracing (g = 0.8) Light tracing (g = 0.9) Light tracing (g = 0)

Figure 7: Importance propagation improves overall radiance distribution across the medium and visibility of bright regions behind. This especially holds for high-albedo media with strong scattering anisotropy (here $g = 0.98$) and when using a low number of ordinates (27 here).

The described sampling scheme can be further improved by considering how much illumination from different directions actually contributes to the image. To this end, we introduce an additional importance propagation step before sampling.

#### Importance propagation

Importance propagation

The described sampling scheme can be further improved by considering how much illumination from different directions actually contributes to the image. To this end, we introduce an additional importance propagation step before sampling.
Figure 9: Workflow of the presented algorithm for a single directional light. For distant environment illumination the volumetric part of the pipeline is very similar, with the exception of rectilinear grids being used to propagate illumination from distant ordinates instead of the combination of VPLs and spherical grids.

Figure 10: In media like clouds the scattering anisotropy plays a significant role in their appearance, thus the common assumption of isotropic scattering prevents a believable rendition of such media. The clouds are rendered by the described method at 12 Hz using 64 ordinates and $20^3$ grid resolution for each of them, with 15 propagation iterations. The scattering anisotropy was set to $g = 0.96$.

us to approximately treat the space as locally Euclidean and obtain plausible results again using virtually the same propagation scheme as before. The main difference in the propagation is that we have to account for the quadratic fall-off: although we base our propagation on radiance, we have to explicitly compensate for the varying grid cell sizes resulting from the non-uniform shell spacing. To this end, we scale the radiance when propagating along the principal direction in proportion to the radial coordinate spacing. A sample demonstration of this propagation type for a point light in a simple homogeneous spherical medium is shown in Fig. 8.

4 RESULTS

All results were computed on a laptop PC with a 2.0 GHz Intel Core i7 CPU, 16 GB of RAM and an NVidia GeForce GTX 485 Mobile card with 2 GB of VRAM. In all our measurements we use the framebuffer resolution of $800 \times 600$ in order to let the computation time be dominated by the propagation rather than ray-marching. Resolutions of the medium density datasets are typically in the order of tens in each dimension (but effectively enhanced by the procedural noise). Although the number of propagation iterations needs to be chosen empirically at the moment, in general we found that amounts similar to the propagation grid resolution along the propagation dimension is sufficient (around 10–20 in our examples). Other specific scene details are provided in the caption of each discussed figure.

We first tested our method for cloudy media with high scattering anisotropy in comparison to their isotropic versions (Fig. 10). It can be seen that our propagation scheme handles both cases well. Interestingly, grid resolutions as well as computation times required to render plausible participating media are rather insensitive to its anisotropy, i.e., anisotropic media render as fast as isotropic media. Although a larger number of ordinates is required to reproduce high-anisotropy effects, this additional effort is usually compensated by a decreased complexity of the spatial radiance distribution, which enables using coarser propagation grids.
We propose a novel discrete ordinates method capable of computing light transport in heterogeneous participating media exhibiting light scattering of virtually arbitrary anisotropy. The method does not require any precomputations, which makes it suitable even for simulating dynamic and evolving media. Our representation also adapts to and prefilters the incident lighting. Radiance is represented by the Heney-Greenstein distribution, and propagated by our novel scheme in volumes oriented along the principal light directions.

In general the steps of the proposed method are physically-plausible (please refer to the supplementary materials for further details). The employed empirical heuristics introduce a certain bias but allow us to make design decisions that result in a near-real time performance on contemporary graphics hardware.

Next, we compare our approach to an unbiased Monte-Carlo reference, as well as SHDOM, in Fig. 11. It is apparent that the described artifacts prevent SHDOM from handling anisotropic media correctly, despite being theoretically capable to do so.

The effect of using different numbers of principal ordinates is shown in Fig. 12. It can be seen that the discretization becomes apparent only with very few ordinates. The importance propagation usually helps to alleviate this by sampling those directions which will influence the solution most significantly. As Fig. 7 demonstrates, this is most likely the opposite side of the medium, suggesting that a simpler empirical heuristic could potentially work in certain cases.

One of the main shortcomings of the importance propagation is its potential temporal incoherence, mostly manifested by temporal flickering. For this reason we filter the importance map both spatially and temporally, which, however, mainly distributes the incoherence over time. One of our main targets for future work is therefore improving this by explicitly enforcing temporal coherency when the sampled light sources relocate due to camera movement.

Prefiltering helps to improve the rendering quality in most scenarios and we used it to generate all results throughout the paper. It is particularly indispensable for media with an optical thickness insufficient to blur the sampled illumination, e.g., as in Fig. 6, where singularity-like artifacts would appear otherwise. Prefiltering removes these artifacts but still allows to perceive features of the background illumination, thanks to its adaptivity.

Finally in general, Fig. 1 and Fig. 13 show our propagation scheme for both regular and radial grids used to render multiple scattering effects in the volume stemming from direct illumination (light shafts), environment lighting, and indirect surface illumination (using virtual local light sources) under fully-dynamic conditions.

5 CONCLUSION

We propose a novel discrete ordinates method capable of computing light transport in heterogeneous participating media exhibiting light scattering of virtually arbitrary anisotropy. The method does not require any precomputations, which makes it suitable even for simulating dynamic and evolving media. Our representation also adapts to and prefilters the incident lighting. Radiance is represented by the Heney-Greenstein distribution, and propagated by our novel scheme in volumes oriented along the principal light directions.

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