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Time Prediction of Mouse-based Cursor Movements

Peter Oel, Paul Schmidt, Alfred Schmitt

Institut für Betriebs- und Dialogsysteme
Universität Karlsruhe
Kaiserstr. 12, 76128 Karlsruhe, Germany
Corresponding author contact: oel@ira.uka.de

SUMMARY

This paper addresses the prediction of movement time of mouse-based cursor transfers between objects in a graphical user interface. We present the data from an experimental study aimed at obtaining an appropriate model for movement time prediction. As can be seen from the data, Fitts' law which is widely used as a predictor for such movements fails with our data. This is not only for low values of Fitts' index of difficulty as known from the literature, but also for small target areas like radio buttons, combo boxes, and buttons on toolbars such as typically occurring with graphical user interfaces. We present a new power model for movement time prediction derived from our data and compare this model to other models from the literature.

KEYWORDS: human-computer interaction, mouse-based pointing tasks, movement time prediction, Fitts' law, power law.

INTRODUCTION

In [9] we presented a method to achieve an optimal button arrangement for a given set of *n* buttons with corresponding absolute probabilities for user induced cursor movements between these buttons. The probabilities reflect the users' behaviour. The position and width of the *n* buttons were determined during an optimization process, where Fitts' law [1] was used to estimate the transfer time for cursor movements. The optimization led to a substantial improvement of the total transfer time for cursor movements compared to non-optimized arrangements of the *n* buttons. Later, when testing these optimized button sets, it turned out that for some sets of buttons the movement time recorded during studies using the optimization lagged behind the theoretical improvements, especially for sets where the buttons lie close together.

Numerous studies in the literature focus on the verification and application of Fitts' law, e.g. [6] and [8]. According to Fitts' law the time *MT* to move to and to select a target of width *W* at a distance *A* is

$$MT = a + b \cdot ID, \quad ID = \log_2(2A/W) \qquad (1)$$

where a and b are empirically determined parameters and *ID* is the so-called index of difficulty. Problems that arise when using Fitts' law with low values of *ID* are well known. Gan and Hoffmann [2] stated that for an *ID* less than 3 the movement is controlled ballistically rather than visually. Within the study for testing the optimization the subjects had to perform an additional experiment to get movement time data for different combinations of *A* and *W*. The experimentally determined data shows an upward curvature of *MT* data away from the linear regression line for low values of *ID*.

The next section describes the experimental conditions, followed by the experimental results. Then we derive our new power model to estimate the transfer time, which is then compared to other known models. Finally we draw some conclusions in the last section.

EXPERIMENTAL CONDITIONS

32 male and female students well familiar with computers had to use a mouse to perform pointing and clicking tasks viewed on a 21" CRT monitor. The monitor had a resolution of 1600 x 1200 pixels. A distance of 20 pixels corresponds to 0,5 cm, however we will only use pixels as the unit of distance. Each single task consisted of a starting area and a target area presented to the subjects on the screen. The subjects had to click in the starting area and then had to move as fast as possible to the target area and perform a second click in the target area. Time was recorded between these two clicks. Subjects were told to click until they hit the target area. Errors were also recorded. Each starting area had a width of 20 pixels. Six different widths $W \in \{10, 20, 40, 80, 60, 120, 320 \text{ pixels}\}$ were used for the target area, at seven distances $A \in \{20, 40, 80, 160, 320, 640, 1280 \text{ pixels}\}$ from the starting area. This results in 32 different tasks, since not all combinations of A and W are possible ($W \leq A$). All areas were of quadratic shape. The direction of movement always remains the same, from left to right, since no significant relevance of movement direction to *MT* was found in our data. The maximum variation of the time needed for different movement directions is less than 7%. Although the differences should be noted, they do not indicate a preference for one particular model. This fact is also reported by [7] and [12].

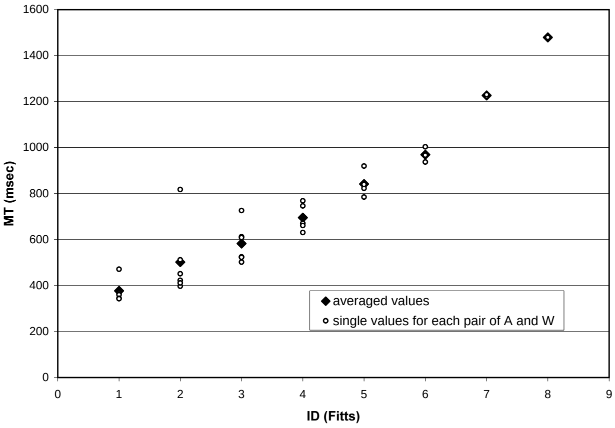

Figure 1: *MT* versus *ID* showing the upwards curvature of the graph for low values of *ID* (large dots) and the wide variation of pairs of *A* and *W* (small circles).

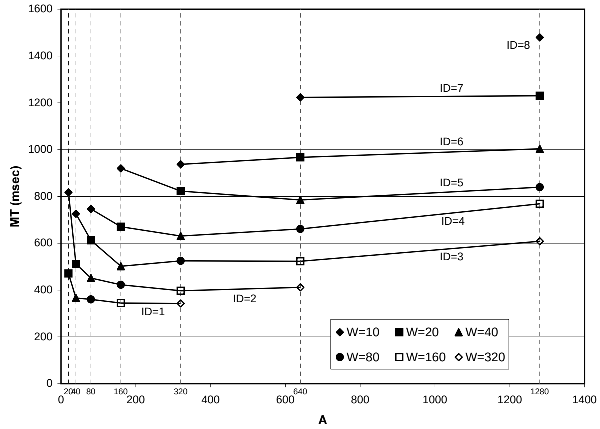

Figure 2: Each pair of *A* and *W* is represented by one symbol. Same symbols correspond to the same width. Problems arise with Fitts' law not only for low *ID*s but also for small targets.

EXPERIMENTAL RESULTS

The average movement time, in milliseconds, for all pairs of distances *A* and widths *W* for the pointing and clicking task and the index of difficulty *ID* according to Fitts are included in Table 1. Incorrect movements, i.e. clicks outside the target area, were not considered.

Figure 1 shows the standard chart for *MT* versus *ID*. The slightly higher values for low *ID*s reported by other researchers can also be found within our data. There is not only a wide distribution of the single movement time values (not displayed in the figure), but also a wide variation of time values for the pairs of *A* and *W* with the same *ID* (displayed as small circles).

To get a different view of our data we re-plot (figure 2) the data using the distance *A* as the unit on the x-axis. Figure 2 also shows all pairs of *A* and *W* using different symbols for each different value of *W*. Dots on the same dashed vertical lines correspond to the same distance *A*. Dots having the same *ID* are connected by lines. According to Fitts' formula all lines of the same *ID* should be horizontal. It is evident that the invariance of Fitts' law against scaling of *A* and W is not given. This is *not* a problem of low *ID*. See the time values for $A \in \{160, 320, 640\}$ in the case of *ID*=1 and *ID*=2, which are nearly constant. Instead it seems that smaller targets need more time to be hit.

Taking these problems into account, we started to look for a better model to get an appropriate time prediction for mouse-based cursor movements.

POWER LAW

First a model is defined which describes the movement time within each class of different *W*s. Every model depending on *A* and using two parameters *h* and *k* could be taken, e.g. $MT=h+k\cdot\log(A)$ or $MT=h\cdot A^k$. The better the model fits the data for each group of *W*s the better the final model will be. The regression results are shown in table 2. The curves are expressed by the two paramters *h* and *k* and the correlation coefficient R^2 for each value of *W*. For each class of *W* the best fit is achieved using the power model $MT=h\cdot A^k$. Figure 3 is a re-draw of figure 2 leaving those *ID* lines away. Instead we present the regression curves of symbols having the identical value of *W*.

Second, we want the factor *h* and the exponent *k* to be dependent on *W* creating a single model, which can be used for all classes of *W*. This leads to

MT | *ID*	*W*=10	*W*=20	*W*=40	*W*=80	*W*=160	*W*=320
A=20	817 | 2	470 | 1				
A=40	726 | 3	511 | 2	366 | 1			
A=80	746 | 4	612 | 3	451 | 2	360 | 1		
A=160	919 | 5	670 | 4	501 | 3	422 | 2	344 | 1	
A=320	937 | 6	823 | 5	630 | 4	524 | 3	397 | 2	342 | 1
A=640	1223 | 7	967 | 6	784 | 5	661 | 4	523 | 3	411 | 2
A=1280	1479 | 8	1230 | 7	1003 | 6	839 | 5	768 | 4	608 | 3

Table 1: Average movement time *MT* (in millisec.) for each pair of distance *A* and target width *W*. The number behind each *MT* value is the index of difficulty according to Fitts.

	power model			log model		
W	*h*	*k*	R^2	*h*	*k*	R^2
10	427,38	0,1571	0,8020	149,64	163,32	0,7815
20	223,83	0,2293	0,9823	-133,77	175,16	0,9320
40	124,93	0,2856	0,9892	-341,57	177,94	0,9465
80	90,32	0,3088	0,9949	-434,86	172,76	0,9659
160	45,46	0,3869	0,9605	-724,72	201,65	0,9121
320	30,39	0,4140	0,9576	-784,64	191,75	0,9276

Table 2: Regression results and correlation coefficient using the power model $MT=h\cdot A^k$ and the logarithmic model $MT=h+k\cdot\log(A)$. The power model is the preferred one.

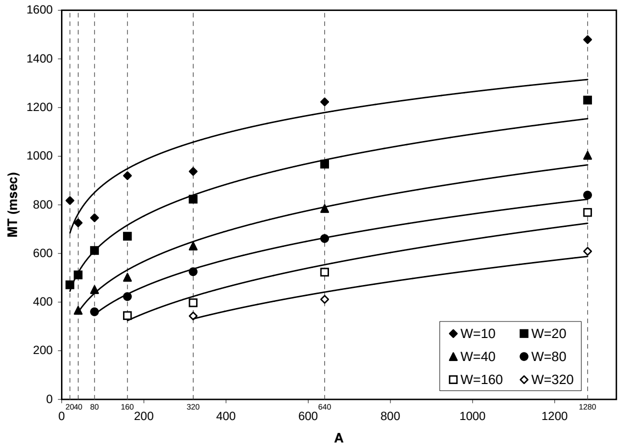

Figure 3: All pairs of *A* and *W*, represented by symbols, and the regression lines for *MT* values corresponding to the same *W* using $MT = h \cdot A^k$ as regression model.

$$MT=h(W)\cdot A^{k(W)}.$$

To get a good approximation for *h(W)* and *k(W)* two regression models must be used which highly fit the values of table 2. In the case of *h(W)* this is $h(W)=a\cdot W^b$. For *k(W)* the best fit is achieved by a logarithmic model $k(W)=c+d\cdot\log_2(W)$. Data, regression lines, regression formulae, and correlation coefficient for the factor and the exponent are presented in figure 4. Correlations for the two models are R^2=0.9927 (factor *h*) and R^2=0.9804 (exponent *k*), which is high enough to get a good fit.

The final power law is given as

$$MT = (a \cdot W^b) \cdot A^{\,c+d\,\cdot\log_2(W)}. \quad (2)$$

It should be mentioned that it is also possible to use $MT=h'\cdot W^{k'}$ as base model for all classes of *A* and to make the factor and exponent be dependent on *A*.

Now we will show, that Fitts' law can be seen as an approximation of the new power law. For a better reading we only will use the logarithm to the natural base *e* in the further text. Base 2 is achieved by a change of coefficients (e.g. $d'=d/\log(2)$). Equation 2 may be expressed in exponential form

$$MT = a \cdot e^{b\cdot\log(W)} \cdot e^{(c+d\cdot\log(W))\cdot\log(A)}$$
$$MT = a \cdot e^{b\cdot\log(W) + c\cdot\log(A) + d\cdot\log(W)\cdot\log(A)}$$

Using the power series expansion $e^x=1+x+1/2!\cdot x^2+1/3!\cdot x^3+\cdots$ we get

$$MT = a + a\cdot(b\cdot\log(W) + c\cdot\log(A) + d\cdot\log(W)\cdot\log(A))$$
$$+ a/2! \cdot (b\cdot\log(W) + c\cdot\log(A) + d\cdot\log(W)\cdot\log(A))^2 + \cdots$$

Fitts' law can be considered as a special case of the new power law. Set $b=-c$, $a=a'+c'\cdot\log(2)$, and $c'=c\cdot a$, to get

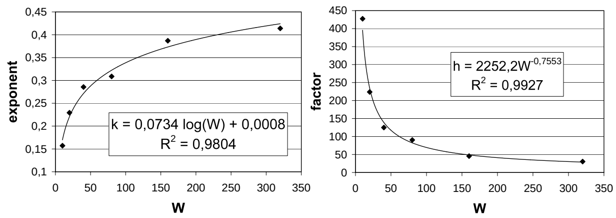

Figure 4: Regression values from table 2 and corresponding regression graphs for the exponent and factor of the power model.

$$MT = a' + c'\cdot\log(2) - c'\cdot\log(W) + c'\cdot\log(A)$$
$$+ a\cdot d\cdot\log(A)\cdot\log(W) + \cdots$$
$$MT = a + c'\cdot\log(2A/W) + a\cdot d\cdot\log(A)\cdot\log(W) + \cdots$$

As can be seen from equation 1 Fitts' law is identical to the first two terms of this power series expansion.

Kvålseth [3] introduced an alternative power model, that he claimed to be an alternative to Fitts' law. The model is given as

$$MT = a \cdot W^b \cdot A^c \quad (3)$$

which can be seen as the first order approximation of our power law. Equation 2 can be transformed to

$$MT = a \cdot W^b \cdot A^c \cdot e^{(d\cdot\log(W)\cdot\log(A))}.$$

The Kvålseth power law is the leading term of this formula when doing a power series expansion for $e^{d\cdot\log(W)\cdot\log(A)}$.

RE-ANALYSIS OF PUBLISHED DATA

In this section we compare our model to some other models reported in the literature. The models taken are Fitts' law (equation 1), our new power law (equation 2), the Kvålseth power law (equation 3), and some other alternative models proposed by

Welford [10]	$MT = a + b\cdot\log_2(A/W+0,5)$	(4)
Welford [11]	$MT = a + b\cdot\log_2(A) + c\cdot\log_2(1/W)$	(5)
Kvålseth [3]	$MT = a\cdot(A/W)^b$	(6)
MacKenzie [4]	$MT = a + b\cdot\log_2(A/W+1)$	(7)

We use the data published in [1], [2], [5], and the results of our study to compare these models. Fitts introduced four sets of data: the reciprocal tapping task with 1 oz. stylus and with 1 lb. stylus, the pin transfer, and the disk transfer task. Gan and Hoffmann provide data of an experiment equivalent to the apparatus used by Fitts. MacKenzie also used Fitts' paradigm except that the subjects used a mouse as input device manipulating a cursor on a CRT display.

Model	E	This study		Gan Hoffm.		MacKenzie		Fitts tap. 1oz		Fitts tap. 1lb		Fitts pin		Fitts disk	
		R^2	G	R^2	G	R^2	G	R^2	G	R^2	G	R^2	G	R^2	G
Power model	2	0,9664	1	0,9195	1	0,9930	1	0,9964	1	0,9954	1	0,9829	1	0,9790	1
Kvålseth	3	0,9331	2	0,8887	2	0,9725	6	0,9889	2	0,9868	2	0,9814	2	0,9639	2
Kvålseth	6	0,9154	3	0,7113	4	0,9704	7	0,9889	3	0,9866	3	0,8514	4	0,9020	4
Welford	5	0,9122	4	0,8577	3	0,9742	4	0,9665	6	0,9598	6	0,9627	3	0,9501	3
MacKenzie	7	0,9011	5	0,6987	5	0,9841	2	0,9873	4	0,9833	4	0,8455	5	0,8934	5
Welford	4	0,8951	6	0,6824	6	0,9812	3	0,9802	5	0,9749	5	0,8447	6	0,8917	6
Fitts	1	0,8839	7	0,6582	7	0,9735	5	0,9664	7	0,9596	7	0,8437	7	0,8896	7

Table 3: Fitting the seven models to the data reported in this paper, the data collected by Fitts, by Gan and Hoffmann, and by MacKenzie. E is the equation number used in this paper, R^2 the correlation coefficient, and G denotes the rank of the model.

The coefficient of multiple determination (R^2) and the rank (G) of the goodness of fit of the models for each of the six data sets can be seen from table 3. Our new power law provides the best fit for all the reported data. What are the reasons for that improvement? Our model uses four parameters, *a*, *b*, *c*, and *d*, which empirically have to be determined during the regression analysis. On the one hand a model using more parameters should have a better fit to the data compared to models using less parameters. On the other hand this is not valid for equation 3 and equation 5. These two models use three parameters and should therefore be in second and third place. Having a closer look at table 3 shows that this reasoning fails. It comes clear, that the good fit is achieved by taking those small target areas into account.

CONCLUSIONS

The reported experiment provides data showing that Fitts' law is not suitable to calculate the transfer time of cursor movements. Small targets need above-average more time to be hit. That is in fact highly relevant for the design of graphical user interfaces, since 10 pixels is typically the size of dialogue elements like a check box or a radio button and 20 pixels is about the size of a button in the toolbar. Besides the well known problem with low IDs, this is the main factor for Fitts' law not predicting the movement time on a graphical mouse-based interface good enough. Taking this into account we showed a technique to derive a new model and presented the new power law that fits the reported data best.

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